UNIVERSITY OF NEW HAVEN

Poisson Kernel Estimates of Lexigraphic Matrices

Jane Q. Student

An Undergraduate Thesis
Submitted to the Department of Mathematics and Physics
of the University of Newhaven

May 2015

ABSTRACT

Poisson Kernel Estimates of Lexigraphic Matrices

by Jane Q. Student

May 2015

In this study we examine the suitability of efficiently inverting, or obtaining the solution of problems associated with a class of centrosymmetric matrices which arise from problems associated with stochastic interpolation on regularly spaced grids. These matrices associated with these linear systems are full, and thus the usual range of sparse matrix solvers are not available. In particular, the study focuses directly on the properties of centrosymmetric matrices as a means to improve the solvability of these linear systems, showing that for these centrosymmetric matrices of even order it is possible to achieve a factor of 7/8 improvement in the cost of matrix inversion using Schur complements. In addition the multiplication of these matrices can be speeded up by a factor of two by fully utilizing the block antisymmetry of these matrices.

The University of New Haven

Poisson Kernel Estimates of Lexigraphic Matrices

by Jane Q. Student

An Undergraduate Thesis
Submitted to the Department of Mathematics and Physics
of the University of Newhaven

Jane Q. Student

Approved by:

Thomas P. Allmath, Advisor

Joseph Kolibal, Department Chair

May 2015

Acknowledgments

I would like to thank my advisor Dr. Strange Strangelove for guidence throughout the project, and Dr. Tom Clown for the help with everything else, and everyone else for financial support.

Dedication

This work is dedicated to my family, especially my mother and father. Their encouragement throughout this process made this a reality.

Contents

List of Figures

List of Tables

Chapter 1

Introduction

1.1 Background

A square matrix has the same number of rows as it does columns, and it represents a linear system in which the number of unknowns exactly matches the number of equation, and thus it represents a linear system which may have a unique solution. The size of an $n \times n$ matrix is referred to as the order of the matrix n, and this equals the number of rows or columns in a square matrix.

Finally, in terms of notation, it will be convenient to refer to the matrix A in terms of its coefficients, using the notation $A = (a_{ij})$. For example, if we multiply the matrix A by the constant c, then $cA = c(a_{ij}) = (ca_{ij})$ and we demonstrate through this notational device that matrix multiplication is equivalent to multiplying each entry in the matrix by the constant c. This standard notational tool will prove useful in regard to demonstrating some properties of matrices that are needed in this study.

1.2 A special matrix of interest: a centrosymmetric matrix

A matrix with a particular structure, or a specific form associated with the pattern of the coefficients is called a *special matrix*. A special matrix need not be complicated, as in the case of the diagonal matrix already discussed, or it can be much more intricate in the layout of its coefficients in some regular pattern. The primary focus of this study is on centrosymmetric matrices, although even at this level of specialization, not every centrosymmetric matrix will be of interest to us. The centrosymmetric matrices that are of interest are those which arise from problems associated with stochastic interpolation.

Centrosymmetric matrices[1] follow a pattern in which the coefficients of the matrix are reflected about the center of the matrix [2], i.e., the coefficients of the matrix A are rotationally symmetric about the center. Alternatively, we can define a centrosymmetric matrix such that $A = EAE$, where $E = (\boldsymbol{e}_n, \boldsymbol{e}_{n-1}, \ldots, \boldsymbol{e}_1)$ is the exchange matrix, and $\boldsymbol{e}_k$ is the k-th standard basis vector in $\mathbb{R}^n$. In this description, the coefficients of the matrix progress

[1] These are sometimes referred to as perplectic matrices.

lexicographically for the first half of the rows in the matrix, and then regress through these same coefficients listed in reverse lexicographic order from the bottom row to the middle row, if the matrix is of even order. If it is of odd order, the middle row is symmetric about its center. Perhaps the easiest way to conceptualize this pattern is through language using palindromes. When read backwards, a palindrome produces the same sentence when read both forwards and backwards as in the sentence, “Go hang a salami, I’m a lasagna hog.”

Lorem ipsum dolor sit amet, consectetuer adipiscing elit. Ut purus elit, vestibulum ut, placerat ac, adipiscing vitae, felis. Curabitur dictum gravida mauris. Nam arcu libero, nonummy eget, consectetuer id, vulputate a, magna. Donec vehicula augue eu neque. Pellentesque habitant morbi tristique senectus et netus et malesuada fames ac turpis egestas. Mauris ut leo. Cras viverra metus rhoncus sem. Nulla et lectus vestibulum urna fringilla ultrices. Phasellus eu tellus sit amet tortor gravida placerat. Integer sapien est, iaculis in, pretium quis, viverra ac, nunc. Praesent eget sem vel leo ultrices bibendum. Aenean faucibus. Morbi dolor nulla, malesuada eu, pulvinar at, mollis ac, nulla. Curabitur auctor semper nulla. Donec varius orci eget risus. Duis nibh mi, congue eu, accumsan eleifend, sagittis quis, diam. Duis eget orci sit amet orci dignissim rutrum.

Nam dui ligula, fringilla a, euismod sodales, sollicitudin vel, wisi. Morbi auctor lorem non justo. Nam lacus libero, pretium at, lobortis vitae, ultricies et, tellus. Donec aliquet, tortor sed accumsan bibendum, erat ligula aliquet magna, vitae ornare odio metus a mi. Morbi ac orci et nisl hendrerit mollis. Suspendisse ut massa. Cras nec ante. Pellentesque a nulla. Cum sociis natoque penatibus et magnis dis parturient montes, nascetur ridiculus mus. Aliquam tincidunt urna. Nulla ullamcorper vestibulum turpis. Pellentesque cursus luctus mauris.

Nulla malesuada porttitor diam. Donec felis erat, congue non, volutpat at, tincidunt tristique, libero. Vivamus viverra fermentum felis. Donec nonummy pellentesque ante. Phasellus adipiscing semper elit. Proin fermentum massa ac quam. Sed diam turpis, molestie vitae, placerat a, molestie nec, leo. Maecenas lacinia. Nam ipsum ligula, eleifend at, accumsan nec, suscipit a, ipsum. Morbi blandit ligula feugiat magna. Nunc eleifend consequat lorem. Sed lacinia nulla vitae enim. Pellentesque tincidunt purus vel magna. Integer non enim. Praesent euismod nunc eu purus. Donec bibendum quam in tellus. Nullam cursus pulvinar lectus. Donec et mi. Nam vulputate metus eu enim. Vestibulum pellentesque felis eu massa.

Quisque ullamcorper placerat ipsum. Cras nibh. Morbi vel justo vitae lacus tincidunt ultrices. Lorem ipsum dolor sit amet, consectetuer adipiscing elit. In hac habitasse platea dictumst. Integer tempus convallis augue. Etiam facilisis. Nunc elementum fermentum wisi. Aenean placerat. Ut imperdiet, enim sed gravida sollicitudin, felis odio placerat quam, ac pulvinar elit purus eget enim. Nunc vitae tortor. Proin tempus nibh sit amet nisl. Vivamus quis tortor vitae risus porta vehicula.

1.3 Developments

Lorem ipsum dolor sit amet, consectetuer adipiscing elit. Ut purus elit, vestibulum ut, placerat ac, adipiscing vitae, felis. Curabitur dictum gravida mauris. Nam arcu libero, nonummy eget, consectetuer id, vulputate a, magna. Donec vehicula augue eu neque. Pellentesque habitant morbi tristique senectus et netus et malesuada fames ac turpis egestas. Mauris ut leo. Cras viverra metus rhoncus sem. Nulla et lectus vestibulum urna fringilla ultrices. Phasellus eu tellus sit amet tortor gravida placerat. Integer sapien est, iaculis in, pretium quis, viverra ac, nunc. Praesent eget sem vel leo ultrices bibendum. Aenean faucibus. Morbi dolor nulla, malesuada eu, pulvinar at, mollis ac, nulla. Curabitur auctor semper nulla. Donec varius orci eget risus. Duis nibh mi, congue eu, accumsan eleifend, sagittis quis, diam. Duis eget orci sit amet orci dignissim rutrum.

Nam dui ligula, fringilla a, euismod sodales, sollicitudin vel, wisi. Morbi auctor lorem non justo. Nam lacus libero, pretium at, lobortis vitae, ultricies et, tellus. Donec aliquet, tortor sed accumsan bibendum, erat ligula aliquet magna, vitae ornare odio metus a mi. Morbi ac orci et nisl hendrerit mollis. Suspendisse ut massa. Cras nec ante. Pellentesque a nulla. Cum sociis natoque penatibus et magnis dis parturient montes, nascetur ridiculus mus. Aliquam tincidunt urna. Nulla ullamcorper vestibulum turpis. Pellentesque cursus luctus mauris.

Nulla malesuada porttitor diam. Donec felis erat, congue non, volutpat at, tincidunt tristique, libero. Vivamus viverra fermentum felis. Donec nonummy pellentesque ante. Phasellus adipiscing semper elit. Proin fermentum massa ac quam. Sed diam turpis, molestie vitae, placerat a, molestie nec, leo. Maecenas lacinia. Nam ipsum ligula, eleifend at, accumsan nec, suscipit a, ipsum. Morbi blandit ligula feugiat magna. Nunc eleifend consequat lorem. Sed lacinia nulla vitae enim. Pellentesque tincidunt purus vel magna. Integer non enim. Praesent euismod nunc eu purus. Donec bibendum quam in tellus. Nullam cursus pulvinar lectus. Donec et mi. Nam vulputate metus eu enim. Vestibulum pellentesque felis eu massa.

Quisque ullamcorper placerat ipsum. Cras nibh. Morbi vel justo vitae lacus tincidunt ultrices. Lorem ipsum dolor sit amet, consectetuer adipiscing elit. In hac habitasse platea dictumst. Integer tempus convallis augue. Etiam facilisis. Nunc elementum fermentum wisi. Aenean placerat. Ut imperdiet, enim sed gravida sollicitudin, felis odio placerat quam, ac pulvinar elit purus eget enim. Nunc vitae tortor. Proin tempus nibh sit amet nisl. Vivamus quis tortor vitae risus porta vehicula.

Fusce mauris. Vestibulum luctus nibh at lectus. Sed bibendum, nulla a faucibus semper, leo velit ultricies tellus, ac venenatis arcu wisi vel nisl. Vestibulum diam. Aliquam pellentesque, augue quis sagittis posuere, turpis lacus congue quam, in hendrerit risus eros eget felis. Maecenas eget erat in sapien mattis porttitor. Vestibulum porttitor. Nulla facilisi. Sed a turpis eu lacus commodo facilisis. Morbi fringilla, wisi in dignissim interdum, justo lectus sagittis dui, et vehicula libero dui cursus dui. Mauris tempor ligula sed lacus. Duis cursus enim ut augue. Cras ac magna. Cras nulla. Nulla egestas. Curabitur a leo. Quisque egestas wisi eget nunc. Nam feugiat lacus vel est. Curabitur consectetuer.

Suspendisse vel felis. Ut lorem lorem, interdum eu, tincidunt sit amet, laoreet vitae, arcu. Aenean faucibus pede eu ante. Praesent enim elit, rutrum at, molestie non, nonummy vel, nisl. Ut lectus eros, malesuada sit amet, fermentum eu, sodales cursus, magna. Donec eu purus. Quisque vehicula, urna sed ultricies auctor, pede lorem egestas dui, et convallis elit erat sed nulla. Donec luctus. Curabitur et nunc. Aliquam dolor odio, commodo pretium, ultricies non, pharetra in, velit. Integer arcu est, nonummy in, fermentum faucibus, egestas vel, odio.

Sed commodo posuere pede. Mauris ut est. Ut quis purus. Sed ac odio. Sed vehicula hendrerit sem. Duis non odio. Morbi ut dui. Sed accumsan risus eget odio. In hac habitasse platea dictumst. Pellentesque non elit. Fusce sed justo eu urna porta tincidunt. Mauris felis odio, sollicitudin sed, volutpat a, ornare ac, erat. Morbi quis dolor. Donec pellentesque, erat ac sagittis semper, nunc dui lobortis purus, quis congue purus metus ultricies tellus. Proin et quam. Class aptent taciti sociosqu ad litora torquent per conubia nostra, per inceptos hymenaeos. Praesent sapien turpis, fermentum vel, eleifend faucibus, vehicula eu, lacus.

Pellentesque habitant morbi tristique senectus et netus et malesuada fames ac turpis egestas. Donec odio elit, dictum in, hendrerit sit amet, egestas sed, leo. Praesent feugiat sapien aliquet odio. Integer vitae justo. Aliquam vestibulum fringilla lorem. Sed neque lectus, consectetuer at, consectetuer sed, eleifend ac, lectus. Nulla facilisi. Pellentesque eget lectus. Proin eu metus. Sed porttitor. In hac habitasse platea dictumst. Suspendisse eu lectus. Ut mi mi, lacinia sit amet, placerat et, mollis vitae, dui. Sed ante tellus, tristique ut, iaculis eu, malesuada ac, dui. Mauris nibh leo, facilisis non, adipiscing quis, ultrices a, dui.

Chapter 2

Methodology

2.1 Introduction to technical writing

This is the introduction section. This should involve a brief discussion of the purpose of the report, what concepts are essential, and what will be investigated. Use this template for writing your analysis files. If you examine this file closely, it has some very useful features in it that will set up your paper so that it has a more professional appearance. You should also review the rules for typesetting LaTeX.

Be mindful of some simple rules in using English:

1. Do not use the first person singular in a formal report. Use the first person plural or the third person in a passive construction. Thus

   I did this work, is unacceptable, while, *We did this work*, or *This work was done*, are acceptable.

2. When typesetting mathematics, use punctuation. All math equations must read as if they were English sentences. You must end these with a period, and put commas where needed.

3. Keep the verb tense constant. Do not switch from past to present, and then back again, and then back again to the present. There may be a need to do this, however be careful when you are doing it.

4. Avoid colloquial English. These are formal reports. These are not email messages to your friends. Be aware that some words in English such as *big* are colloquial, and do not appear in formal writing. Do not use *very* to modify every adjective. It is usually meaningless.

   Get a manual on English usage and style from the library, or purchase one from the bookstore. Realize that if you plan to stay in academia, or get an administrative position in a company or a laboratory, you have to be able to communicate effectively. Work on this skill.

Then be mindful of some simple rules for typesetting LaTeX:

1. Note that the previous sentence beginning this list is not indented since it does not begin a new paragraph. To avoid indentation, use the `\noindent` command.

2. Note the `\usepackage` command which changes the font and invokes the AMS mathematics commands (which simplify typesetting). Note the use of the

   `\thispagestyle{empty}`

   command to avoid the printing of page number on the title page.

3. Use LaTeXcommands carefully. Please review how to use math mode. Many of the reports had math in the text that was not in math mode. All symbols, variable, and so on are to be written in math mode. We do not use computer science variables to write equations, either, thus

   $$\sum_{i=1}^{upperbound} x_i^2$$

   is not acceptable, while

   $$\sum_{i=1}^{n} x_i^2$$

   is acceptable.

4. There is no need for asterices in multiplication. Thus ab or xy, not $a*b$ or $x*y$.

5. Please note that LaTeXuses the forward and backward quotes. Thus we write *“This is a quote”*, and not *”This is a quote”*. Examine this example closely. The forward quote is written using the ‘ and not the ’ quote. The forward quote is typically on the upper left of your keyboard, on the tilde key, and below the Esc key.

2.1.1 Background on technical writing

Introduce the mathematical ideas and how they are implemented in this section. You should not have any results here. Only introduce the mathematics that is necessary to discuss your results. For example, the method used to compute each norm may be important, however you may choose to either typeset the equations or refer to these in your text, or any other reference. You may refer to equations using a bibliography reference generated using a bib file.

2.1.2 What’s in the Results

Discuss the results. This means that you provide details concerning the numerical experiments that were performed. This section should include graphs, tables and other supporting documentation. It should read well, and all items discussed in the text should refer to tables, graphs and equations using the reference command in LaTeXand using the format `Table~\ref{table:mytable}`, `Figure~\ref{figure:myfigure}` to refer to tables and figures. Equations should be referred to using the number of the equation in parentheses, i.e., `(\ref{eq:myequation})`. You never use the word 'equation', except at the beginning of a sentence, in which case you use the construction: `Eq.~(\ref{eq:myequation})`. Note that you need the ~, i.e., the tilde, to avoid having LaTeXbreak the object across a line. Always tie counters to their attributes using a tilde or a thin space, e.g.,

tied using a tilde: `in Sec.~\ref{sec:topics}` $\Longrightarrow$ in Sec. 3.7
tied using a thin space: `in Sec.\,\ref{sec:topics}` $\Longrightarrow$ in Sec. 3.7
untied (wrong): `in Sec. \ref{sec:topics}` $\Longrightarrow$ in Sec. 3.7

which ties the object Sec. to the counter. Not only is the space too large after the period when there is no tie, but there is the danger that the word, Sec., will go at the end of one line, and the number 3.7 at the start of a new line.

When doing numerical studies, you must present the results of all of your numerical investigations. For the case of polynomial interpolation, at the very least these results should have examined the behaviour of the polynomials interpolated on 2^k points, where $k = 2, 3, \ldots, n$, where n is sufficiently large so as to be able to determine numerically whether convergence was achieved. A value of $n = 5$, or $n = 10$ is usually insufficient.

Focus on the error. Numerical methods are approximations. They introduce error. Examine the errors, and attempt to understand where they are coming from. Examine issues of computational efficiency, and other topics as appropriate. Time your results when you run the codes. You can always choose not to use the timing results, however if they show something important, then you can use it.

2.1.3 What's in the Conclusion

Be brief, however one sentence is not sufficient. Try to concisely summarize what was discussed in the results. Avoid rewriting the results section. Someone reading your report should be able to understand the writeup from what is written in this section. This is one of the most important parts of your report (and probably the only one your supervisor or manager will ever read, so you might as well learn to write it effectively).

2.1.4 Using an Appendix

If there are details concerning the code which you wrote which are significant and pertain to the discussion, place these here. While we are on the topic of codes and software, please put your code variables into \tt font, e.g.,

Acceptable style:

We used the value of `NumMax` to determine the degree of the polynomial ...

Unacceptable style:

We used the value of **NumMax** to determine the degree of the polynomial ...

Write all code segments that are discussed using the same method, and use the verbatim environment, i.e.,

```
ofstream outfile;
outfile.open(names,ios::app);
```

as this will prevent the code from being typeset as ordinary text. There are special ways of inputing code that also can be used, and these are described in detail in the appendices. We can link to these conveniently within LaTeX using reference and label commands. For example, we can go to Appendix A, Appendix B, Appendix C, or Appendix D using the \label{} command to label an environment and using the \ref{} command to refer to it in the text.

As an ending note, use the math-preamble.tex file to define commonly used math objects. For example, in typing $\frac{3}{4}$ we did not have to enter math mode to get the fraction displayed as we used the \ensuremath command to ensure that it was in math mode. The user's ability to define macros, such as

```
\newcommand{\vfrac}[2]{\ensuremath{\frac{#1}{#2}}}
```

is one of the enormous strengths of LaTeX. Instead of constantly having to type

```
\ensuremath{\frac{a}{b}}
```

the user can now type

```
\vrac{a}{b}
```

to get the same result, $\frac{a}{b}$.

Chapter 3

Working with fonts

3.1 Some Useful Fonts

These symbols, like ¶, §, †, and ‡, can also be used in math mode, and will change sizes correctly in subscripts and superscripts. The Blackboard Bold letters $\mathbb{A}, \ldots, \mathbb{Z}$ can be accessed by typing (in math mode). Additional symbols of interest include ■, ►, ★, and ✓. To get bold in math use command boldsymbol, $x, y, f(x)$.

Signs that text is the medium of the page. In the example used to start a page with a large letter S, the example illustrates the use of the kern command and the font command to select a font directly with a scaling factor:

Some other fonts of utility include he fractur fonts, which can be called directly using TEX commands (in various sizes):

A B C D E F G H I J K L M N O P Q R S T U V W X Y Z
A B C D E F G H I J K L M N O P Q R S T U V W X Y Z
A B C D E F G H I J K L M N O P Q R S T U V W X Y Z
A B C D E F G H I J K L M N O P Q R S T U V W X Y Z
a b c d e f g h i j k l m n o p q r ſ t u v w x y z

The color for the dropdown font is chosen with the `\textcolor{}` command. There are quite a range of colors that can be accessed, as described in Appendix *D*. They can also be custom constructed, i.e., there is no practical limit on the color range.

Letters in the eufm font can be accessed (in math mode) by typing, for example, `\frak A` or `frak g` to get $\mathfrak{A}\mathfrak{g}$. For the other Euler fonts, see the various "Euler" subsections under the sections for different macro packages (LATEX, plain TEX). The math calligraphic letters are modified from the standard ones in latex by using the `eucal.sty` package. These yield, $\mathcal{ABCDEFGHIJKLMNOPQRSTUVWXYZ}$.

Do not over do the fonts. If there are too many font changes, the text becomes difficult to read, however, some worthwhile font selections include the use of the small caps font to head a small section:

3.2 Scaling fonts in math

Scaling integrals:

- The standard way to scale a font of an image, is to use the `\scalebox` command which is part of the `graphicx` package, using for example

  ```
  {\large E\scalebox{0.7}{DUCATION}}
  ```

  to get the letter E to be larger than the other letters in the word,

  EDUCATION

  This does not work in math mode, though. In math mode there are techniques available for scaling the size of the text that is displayed. The most common of thees is to use the command `\displaystyle` to put math into display mode. For example:

- The standard integral is given by:

  $\int_a^b f(x)dx.$

- The integral using `\displaystyle`:

  $\displaystyle\int_a^b f(x)dx.$

- The integral using `\displaystyle` with the `\mathlarger` command from the `relsize` package:

  $$\int_a^b f(x)dx.$$

- The integral using `\displaystyle` with the `\bigintsss` command from the `bigint` package:

  $$\int_a^b f(x)dx.$$

- The integral using `\displaystyle` with the `\bigintss` command from the `bigint` package:

  $$\int_a^b f(x)dx.$$

- The integral using `\displaystyle` with the `\bigints` command from the `bigint` package:

  $$\int_a^b f(x)dx.$$

- The integral using `\displaystyle` with the `\bigint` command from the `bigint` package:

  $$\int_a^b f(x)dx.$$

- You can of course, place the limits on top and bottom of the integral using the `\limits` command, however it will have to be adjusted for the `\bingint` type commands:

  $$\int\limits_a^b f(x)dx, \qquad \int\limits_a^b f(x)dx.$$

3.3 Using different fonts

This section explores setting fonts dynamically using the

```
\usefont{T1}{FAMILY}{m}{n}\fontsize{F}{S}\selectfont
```

command, where F is the font size and S is the font spacing, and FAMILY is the code for the font family. In these examples, the font family and the font name are give before some text generated using that font family.

ptm Times

Lorem ipsum dolor sit amet, consectetuer adipiscing elit. Ut purus elit, vestibulum ut, placerat ac, adipiscing vitae, felis. Curabitur dictum gravida mauris. Nam arcu libero, nonummy eget, consectetuer id, vulputate a, magna. Donec vehicula augue eu neque. Pellentesque habitant morbi tristique senectus et netus et malesuada fames ac turpis egestas. Mauris ut leo. Cras viverra metus rhoncus sem. Nulla et lectus vestibulum urna fringilla ultrices. Phasellus eu tellus sit amet tortor gravida placerat. Integer sapien est, iaculis in, pretium quis, viverra ac, nunc. Praesent eget sem vel leo ultrices bibendum. Aenean faucibus. Morbi dolor nulla, malesuada eu, pulvinar at, mollis ac, nulla. Curabitur auctor semper nulla. Donec varius orci eget risus. Duis nibh mi, congue eu, accumsan eleifend, sagittis quis, diam. Duis eget orci sit amet orci dignissim rutrum.

lmss Latin Modern Sans

Lorem ipsum dolor sit amet, consectetuer adipiscing elit. Ut purus elit, vestibulum ut, placerat ac, adipiscing vitae, felis. Curabitur dictum gravida mauris. Nam arcu libero, nonummy eget, consectetuer id, vulputate a, magna. Donec vehicula augue eu neque. Pellentesque habitant morbi tristique senectus et netus et malesuada fames ac turpis egestas. Mauris ut leo. Cras viverra metus rhoncus sem. Nulla et lectus vestibulum urna fringilla ultrices. Phasellus eu tellus sit amet tortor gravida placerat. Integer sapien est, iaculis in, pretium quis, viverra ac, nunc. Praesent eget sem vel leo ultrices bibendum. Aenean faucibus. Morbi dolor nulla, malesuada eu, pulvinar at, mollis ac, nulla. Curabitur auctor semper nulla. Donec varius orci eget risus. Duis nibh mi, congue eu, accumsan eleifend, sagittis quis, diam. Duis eget orci sit amet orci dignissim rutrum.

lmr Latin Modern

Lorem ipsum dolor sit amet, consectetuer adipiscing elit. Ut purus elit, vestibulum ut, placerat ac, adipiscing vitae, felis. Curabitur dictum gravida mauris. Nam arcu libero, nonummy eget, consectetuer id, vulputate a, magna. Donec vehicula augue eu neque. Pellentesque habitant morbi tristique senectus et netus et malesuada fames ac turpis egestas. Mauris ut leo. Cras viverra metus rhoncus sem. Nulla et lectus vestibulum urna fringilla ultrices. Phasellus eu tellus sit amet tortor gravida placerat. Integer sapien est, iaculis in, pretium quis, viverra ac, nunc. Praesent eget sem vel leo ultrices bibendum. Aenean faucibus. Morbi dolor nulla, malesuada eu, pulvinar at, mollis ac, nulla. Curabitur auctor semper nulla. Donec varius orci eget risus. Duis nibh mi, congue eu, accumsan eleifend, sagittis quis, diam. Duis eget orci sit amet orci dignissim rutrum.

ccr Computer Concrete

Lorem ipsum dolor sit amet, consectetuer adipiscing elit. Ut purus elit, vestibulum ut, placerat ac, adipiscing vitae, felis. Curabitur dictum gravida mauris. Nam arcu libero, nonummy eget, consectetuer id, vulputate a, magna. Donec vehicula augue eu neque. Pellentesque habitant morbi tristique senectus et netus et malesuada fames ac turpis egestas. Mauris ut leo. Cras viverra metus rhoncus sem. Nulla et lectus vestibulum urna fringilla ultrices. Phasellus eu tellus sit amet tortor gravida placerat. Integer sapien est, iaculis in, pretium quis, viverra ac, nunc. Praesent eget sem vel leo ultrices bibendum. Aenean faucibus. Morbi dolor nulla, malesuada eu, pulvinar at, mollis ac, nulla. Curabitur auctor semper nulla. Donec varius orci eget risus. Duis nibh mi, congue eu, accumsan eleifend, sagittis quis, diam. Duis eget orci sit amet orci dignissim rutrum.

lmtt Latin Modern Typewriter

Lorem ipsum dolor sit amet, consectetuer adipiscing elit. Ut purus elit, vestibulum ut, placerat ac, adipiscing vitae, felis. Curabitur dictum gravida mauris. Nam arcu libero, nonummy eget, consectetuer id, vulputate a, magna. Donec vehicula augue eu neque. Pellentesque habitant morbi tristique senectus et netus et malesuada fames ac turpis egestas. Mauris ut leo. Cras viverra metus rhoncus sem. Nulla et lectus vestibulum urna fringilla ultrices. Phasellus eu tellus sit amet tortor gravida placerat. Integer sapien est, iaculis in, pretium quis, viverra ac, nunc. Praesent eget sem vel leo ultrices bibendum. Aenean faucibus. Morbi dolor nulla, malesuada eu, pulvinar at, mollis ac, nulla. Curabitur auctor semper nulla. Donec varius orci eget risus. Duis nibh mi, congue eu, accumsan eleifend, sagittis quis, diam. Duis eget orci sit amet orci dignissim rutrum.

uncl Uncial

Lorem ipsum dolor sit amet, consectetuer adipiscing elit. Ut purus elit, vestibulum ut, placerat ac, adipiscing vitae, felis. Curabitur dictum gravida mauris. Nam arcu libero, nonummy eget, consectetuer id, vulputate a, magna. Donec vehicula augue eu neque. Pellentesque habitant morbi tristique senectus et netus et malesuada fames ac turpis egestas. Mauris ut leo. Cras viverra metus rhoncus sem. Nulla et lectus vestibulum urna fringilla ultrices. Phasellus eu tellus sit amet tortor gravida placerat. Integer sapien est, iaculis in, pretium quis, viverra ac, nunc. Praesent eget sem vel leo ultrices bibendum. Aenean faucibus. Morbi dolor nulla, malesuada eu, pulvinar at, mollis ac, nulla. Curabitur auctor semper nulla. Donec varius orci eget risus. Duis nibh mi, congue eu, accumsan eleifend, sagittis quis, diam. Duis eget orci sit amet orci dignissim rutrum.

pzc Zapf Chancery

Lorem ipsum dolor sit amet, consectetuer adipiscing elit. Ut purus elit, vestibulum ut, placerat ac, adipiscing vitae, felis. Curabitur dictum gravida mauris. Nam arcu libero, nonummy eget, consectetuer id, vulputate a, magna. Donec vehicula augue eu neque. Pellentesque habitant morbi tristique senectus et netus et malesuada fames ac turpis egestas. Mauris ut leo. Cras viverra metus rhoncus sem. Nulla et lectus vestibulum urna fringilla ultrices. Phasellus eu tellus sit amet tortor gravida placerat. Integer sapien est, iaculis in, pretium quis, viverra ac, nunc. Praesent eget sem vel leo ultrices bibendum. Aenean faucibus. Morbi dolor nulla, malesuada eu, pulvinar at, mollis ac, nulla. Curabitur auctor semper nulla. Donec varius orci eget risus. Duis nibh mi, congue eu, accumsan eleifend, sagittis quis, diam. Duis eget orci sit amet orci dignissim rutrum.

phv Helvetica

Lorem ipsum dolor sit amet, consectetuer adipiscing elit. Ut purus elit, vestibulum ut, placerat ac, adipiscing vitae, felis. Curabitur dictum gravida mauris. Nam arcu libero, nonummy eget, consectetuer id, vulputate a, magna. Donec vehicula augue eu neque. Pellentesque habitant morbi tristique senectus et netus et malesuada fames ac turpis egestas. Mauris ut leo. Cras viverra metus rhoncus sem. Nulla et lectus vestibulum urna fringilla ultrices. Phasellus eu tellus sit amet tortor gravida placerat. Integer sapien est, iaculis in, pretium quis, viverra ac, nunc. Praesent eget sem vel leo ultrices bibendum. Aenean faucibus. Morbi dolor nulla, malesuada eu, pulvinar at, mollis ac, nulla. Curabitur auctor semper nulla. Donec varius orci eget risus. Duis nibh mi, congue eu, accumsan eleifend, sagittis quis, diam. Duis eget orci sit amet orci dignissim rutrum.

fvs Bitstream Vera Sans

Lorem ipsum dolor sit amet, consectetuer adipiscing elit. Ut purus elit, vestibulum ut, placerat ac, adipiscing vitae, felis. Curabitur dictum gravida mauris. Nam arcu libero, nonummy eget, consectetuer id, vulputate a, magna. Donec vehicula augue eu neque. Pellentesque habitant morbi tristique senectus et netus et malesuada fames ac turpis egestas. Mauris ut leo. Cras viverra metus rhoncus sem. Nulla et lectus vestibulum urna fringilla ultrices. Phasellus eu tellus sit amet tortor gravida placerat. Integer sapien est, iaculis in, pretium quis, viverra ac, nunc. Praesent eget sem vel leo ultrices bibendum. Aenean faucibus. Morbi dolor nulla, malesuada eu, pulvinar at, mollis ac, nulla. Curabitur auctor semper nulla. Donec varius orci eget risus. Duis nibh mi, congue eu, accumsan eleifend, sagittis quis, diam. Duis eget orci sit amet orci dignissim rutrum.

ptm Times

Lorem ipsum dolor sit amet, consectetuer adipiscing elit. Ut purus elit, vestibulum ut, placerat ac, adipiscing vitae, felis. Curabitur dictum gravida mauris. Nam arcu libero, nonummy eget, consectetuer id, vulputate a, magna. Donec vehicula augue eu neque. Pellentesque habitant morbi tristique senectus et netus et malesuada fames ac turpis egestas. Mauris ut leo. Cras viverra metus rhoncus sem. Nulla et lectus vestibulum urna fringilla ultrices. Phasellus eu tellus sit amet tortor gravida placerat. Integer sapien est, iaculis in, pretium quis, viverra ac, nunc. Praesent eget sem vel leo ultrices bibendum. Aenean faucibus. Morbi dolor nulla, malesuada eu, pulvinar at, mollis ac, nulla. Curabitur auctor semper nulla. Donec varius orci eget risus. Duis nibh mi, congue eu, accumsan eleifend, sagittis quis, diam. Duis eget orci sit amet orci dignissim rutrum.

LinuxLibertineT-OsF

Lorem ipsum dolor sit amet, consectetuer adipiscing elit. Ut purus elit, vestibulum ut, placerat ac, adipiscing vitae, felis. Curabitur dictum gravida mauris. Nam arcu libero, nonummy eget, consectetuer id, vulputate a, magna. Donec vehicula augue eu neque. Pellentesque habitant morbi tristique senectus et netus et malesuada fames ac turpis egestas. Mauris ut leo. Cras viverra metus rhoncus sem. Nulla et lectus vestibulum urna fringilla ultrices. Phasellus eu tellus sit amet tortor gravida placerat. Integer sapien est, iaculis in, pretium quis, viverra ac, nunc. Praesent eget sem vel leo ultrices bibendum. Aenean faucibus. Morbi dolor nulla, malesuada eu, pulvinar at, mollis ac, nulla. Curabitur auctor semper nulla. Donec varius orci eget risus. Duis nibh mi, congue eu, accumsan eleifend, sagittis quis, diam. Duis eget orci sit amet orci dignissim rutrum.

LinuxBiolinumT-OsF

Lorem ipsum dolor sit amet, consectetuer adipiscing elit. Ut purus elit, vestibulum ut, placerat ac, adipiscing vitae, felis. Curabitur dictum gravida mauris. Nam arcu libero, nonummy eget, consectetuer id, vulputate a, magna. Donec vehicula augue eu neque. Pellentesque habitant morbi tristique senectus et netus et malesuada fames ac turpis egestas. Mauris ut leo. Cras viverra metus rhoncus sem. Nulla et lectus vestibulum urna fringilla ultrices. Phasellus eu tellus sit amet tortor gravida placerat. Integer sapien est, iaculis in, pretium quis, viverra ac, nunc. Praesent eget sem vel leo ultrices bibendum. Aenean faucibus. Morbi dolor nulla, malesuada eu, pulvinar at, mollis ac, nulla. Curabitur auctor semper nulla. Donec varius orci eget risus. Duis nibh mi, congue eu, accumsan eleifend, sagittis quis, diam. Duis eget orci sit amet orci dignissim rutrum.

LinuxBiolinumT-OsF

Lorem ipsum dolor sit amet, consectetuer adipiscing elit. Ut purus elit, vestibulum ut, placerat ac, adipiscing vitae, felis. Curabitur dictum gravida mauris. Nam arcu libero, nonummy eget, consectetuer id, vulputate a, magna. Donec vehicula augue eu neque. Pellentesque habitant morbi tristique senectus et netus et malesuada fames ac turpis egestas. Mauris ut leo. Cras viverra metus rhoncus sem. Nulla et lectus vestibulum urna fringilla ultrices. Phasellus eu tellus sit amet tortor gravida placerat. Integer sapien est, iaculis in, pretium quis, viverra ac, nunc. Praesent eget sem vel leo ultrices bibendum. Aenean faucibus. Morbi dolor nulla, malesuada eu, pulvinar at, mollis ac, nulla. Curabitur auctor semper nulla. Donec varius orci eget risus. Duis nibh mi, congue eu, accumsan eleifend, sagittis quis, diam. Duis eget orci sit amet orci dignissim rutrum.

pag Avant Garde

Lorem ipsum dolor sit amet, consectetuer adipiscing elit. Ut purus elit, vestibulum ut, placerat ac, adipiscing vitae, felis. Curabitur dictum gravida mauris. Nam arcu libero, nonummy eget, consectetuer id, vulputate a, magna. Donec vehicula augue eu neque. Pellentesque habitant morbi tristique senectus et netus et malesuada fames ac turpis egestas.

Mauris ut leo. Cras viverra metus rhoncus sem. Nulla et lectus vestibulum urna fringilla ultrices. Phasellus eu tellus sit amet tortor gravida placerat. Integer sapien est, iaculis in, pretium quis, viverra ac, nunc. Praesent eget sem vel leo ultrices bibendum. Aenean faucibus. Morbi dolor nulla, malesuada eu, pulvinar at, mollis ac, nulla. Curabitur auctor semper nulla. Donec varius orci eget risus. Duis nibh mi, congue eu, accumsan eleifend, sagittis quis, diam. Duis eget orci sit amet orci dignissim rutrum.

pbk Bookman

Lorem ipsum dolor sit amet, consectetuer adipiscing elit. Ut purus elit, vestibulum ut, placerat ac, adipiscing vitae, felis. Curabitur dictum gravida mauris. Nam arcu libero, nonummy eget, consectetuer id, vulputate a, magna. Donec vehicula augue eu neque. Pellentesque habitant morbi tristique senectus et netus et malesuada fames ac turpis egestas. Mauris ut leo. Cras viverra metus rhoncus sem. Nulla et lectus vestibulum urna fringilla ultrices. Phasellus eu tellus sit amet tortor gravida placerat. Integer sapien est, iaculis in, pretium quis, viverra ac, nunc. Praesent eget sem vel leo ultrices bibendum. Aenean faucibus. Morbi dolor nulla, malesuada eu, pulvinar at, mollis ac, nulla. Curabitur auctor semper nulla. Donec varius orci eget risus. Duis nibh mi, congue eu, accumsan eleifend, sagittis quis, diam. Duis eget orci sit amet orci dignissim rutrum.

bch Charter

Lorem ipsum dolor sit amet, consectetuer adipiscing elit. Ut purus elit, vestibulum ut, placerat ac, adipiscing vitae, felis. Curabitur dictum gravida mauris. Nam arcu libero, nonummy eget, consectetuer id, vulputate a, magna. Donec vehicula augue eu neque. Pellentesque habitant morbi tristique senectus et netus et malesuada fames ac turpis egestas. Mauris ut leo. Cras viverra metus rhoncus sem. Nulla et lectus vestibulum urna fringilla ultrices. Phasellus eu tellus sit amet tortor gravida placerat. Integer sapien est, iaculis in, pretium quis, viverra ac, nunc. Praesent eget sem vel leo ultrices bibendum. Aenean faucibus. Morbi dolor nulla, malesuada eu, pulvinar at, mollis ac, nulla. Curabitur auctor semper nulla. Donec varius orci eget risus. Duis nibh mi, congue eu, accumsan eleifend, sagittis quis, diam. Duis eget orci sit amet orci dignissim rutrum.

put Utopia

Lorem ipsum dolor sit amet, consectetuer adipiscing elit. Ut purus elit, vestibulum ut, placerat ac, adipiscing vitae, felis. Curabitur dictum gravida mauris. Nam arcu libero, nonummy eget, consectetuer id, vulputate a, magna. Donec vehicula augue eu neque. Pellentesque habitant morbi tristique senectus et netus et malesuada fames ac turpis egestas. Mauris ut leo. Cras viverra metus rhoncus sem. Nulla et lectus vestibulum urna fringilla ultrices. Phasellus eu tellus sit amet tortor gravida placerat. Integer sapien est, iaculis in, pretium quis, viverra ac, nunc. Praesent eget sem vel leo ultrices bibendum. Aenean faucibus. Morbi dolor nulla, malesuada eu, pulvinar at, mollis ac, nulla. Curabitur auctor semper nulla. Donec varius orci eget risus. Duis nibh mi, congue eu, accumsan eleifend, sagittis quis, diam. Duis eget orci sit amet orci dignissim rutrum.

pnc New Century Schoolbook

Lorem ipsum dolor sit amet, consectetuer adipiscing elit. Ut purus elit, vestibulum ut, placerat ac, adipiscing vitae, felis. Curabitur dictum gravida mauris. Nam arcu libero, nonummy eget, consectetuer id, vulputate a, magna. Donec vehicula augue eu neque. Pellentesque habitant morbi tristique senectus et netus et malesuada fames ac turpis egestas. Mauris ut leo. Cras viverra metus rhoncus sem. Nulla et lectus vestibulum urna fringilla ultrices. Phasellus eu tellus sit amet tortor gravida placerat. Integer sapien est, iaculis in, pretium quis, viverra ac, nunc. Praesent eget sem vel leo ultrices bibendum. Aenean faucibus. Morbi dolor nulla, malesuada eu, pulvinar at, mollis ac, nulla. Curabitur auctor semper nulla. Donec varius orci eget risus. Duis nibh mi, congue eu, accumsan eleifend, sagittis quis, diam. Duis eget orci sit amet orci dignissim rutrum.

enc ENC

Lorem ipsum dolor sit amet, consectetuer adipiscing elit. Ut purus elit, vestibulum ut, placerat ac, adipiscing vitae, felis. Curabitur dictum gravida mauris. Nam arcu libero, nonummy eget, consectetuer id, vulputate a, magna. Donec vehicula augue eu neque. Pellentesque habitant morbi tristique senectus et netus et malesuada fames ac turpis egestas. Mauris ut leo. Cras viverra metus rhoncus sem. Nulla et lectus vestibulum urna fringilla ultrices. Phasellus eu tellus sit amet tortor gravida placerat. Integer sapien est, iaculis in, pretium quis, viverra ac, nunc. Praesent eget sem vel leo ultrices bibendum. Aenean faucibus. Morbi dolor nulla, malesuada eu, pulvinar at, mollis ac, nulla. Curabitur auctor semper nulla. Donec varius orci eget risus. Duis nibh mi, congue eu, accumsan eleifend, sagittis quis, diam. Duis eget orci sit amet orci dignissim rutrum.

qag Gyre Adventor

Lorem ipsum dolor sit amet, consectetuer adipiscing elit. Ut purus elit, vestibulum ut, placerat ac, adipiscing vitae, felis. Curabitur dictum gravida mauris. Nam arcu libero, nonummy eget, consectetuer id, vulputate a, magna. Donec vehicula augue eu neque. Pellentesque habitant morbi tristique senectus et netus et malesuada fames ac turpis egestas. Mauris ut leo. Cras viverra metus rhoncus sem. Nulla et lectus vestibulum urna fringilla ultrices. Phasellus eu tellus sit amet tortor gravida placerat. Integer sapien est, iaculis in, pretium quis, viverra ac, nunc. Praesent eget sem vel leo ultrices bibendum. Aenean faucibus. Morbi dolor nulla, malesuada eu, pulvinar at, mollis ac, nulla. Curabitur auctor semper nulla. Donec varius orci eget risus. Duis nibh mi, congue eu, accumsan eleifend, sagittis quis, diam. Duis eget orci sit amet orci dignissim rutrum.

qpl Gyre Pagella

Lorem ipsum dolor sit amet, consectetuer adipiscing elit. Ut purus elit, vestibulum ut, placerat ac, adipiscing vitae, felis. Curabitur dictum gravida mauris. Nam arcu libero, nonummy eget, consectetuer id, vulputate a, magna. Donec vehicula augue eu neque. Pellentesque habitant morbi tristique senectus et netus et malesuada fames ac turpis egestas. Mauris ut leo. Cras viverra metus rhoncus sem. Nulla et lectus vestibulum urna fringilla ultrices. Phasellus eu tellus sit amet tortor gravida placerat. Integer sapien est, iaculis in, pretium quis, viverra ac, nunc. Praesent eget sem vel leo ultrices bibendum. Aenean faucibus. Morbi dolor nulla, malesuada eu, pulvinar at, mollis ac, nulla. Curabitur auctor semper nulla. Donec varius orci eget risus. Duis nibh mi, congue eu, accumsan eleifend, sagittis quis, diam. Duis eget orci sit amet orci dignissim rutrum.

qhv Gyre Heros

Lorem ipsum dolor sit amet, consectetuer adipiscing elit. Ut purus elit, vestibulum ut, placerat ac, adipiscing vitae, felis. Curabitur dictum gravida mauris. Nam arcu libero, nonummy eget, consectetuer id, vulputate a, magna. Donec vehicula augue eu neque. Pellentesque habitant morbi tristique senectus et netus et malesuada fames ac turpis egestas. Mauris ut leo. Cras viverra metus rhoncus sem. Nulla et lectus vestibulum urna fringilla ultrices. Phasellus eu tellus sit amet tortor gravida placerat. Integer sapien est, iaculis in, pretium quis, viverra ac, nunc. Praesent eget sem vel leo ultrices bibendum. Aenean faucibus. Morbi dolor nulla, malesuada eu, pulvinar at, mollis ac, nulla. Curabitur auctor semper nulla. Donec varius orci eget risus. Duis nibh mi, congue eu, accumsan eleifend, sagittis quis, diam. Duis eget orci sit amet orci dignissim rutrum.

ppl Palatino

Lorem ipsum dolor sit amet, consectetuer adipiscing elit. Ut purus elit, vestibulum ut, placerat ac, adipiscing vitae, felis. Curabitur dictum gravida mauris. Nam arcu libero, nonummy eget, consectetuer id, vulputate a, magna. Donec vehicula augue eu neque. Pellentesque habitant morbi tristique senectus et netus et malesuada fames ac turpis egestas. Mauris ut leo. Cras viverra metus rhoncus sem. Nulla et lectus vestibulum urna fringilla ultrices. Phasellus eu tellus sit amet tortor gravida placerat. Integer sapien est, iaculis in, pretium quis, viverra ac, nunc. Praesent eget sem vel leo ultrices bibendum. Aenean faucibus. Morbi dolor nulla, malesuada eu, pulvinar at, mollis ac, nulla. Curabitur auctor semper nulla. Donec varius orci eget risus. Duis nibh mi, congue eu, accumsan eleifend, sagittis quis, diam. Duis eget orci sit amet orci dignissim rutrum.

PTSansNarrow-TLF PT Sans Narrow

Lorem ipsum dolor sit amet, consectetuer adipiscing elit. Ut purus elit, vestibulum ut, placerat ac, adipiscing vitae, felis. Curabitur dictum gravida mauris. Nam arcu libero, nonummy eget, consectetuer id, vulputate a, magna. Donec vehicula augue eu neque. Pellentesque habitant morbi tristique senectus et netus et malesuada fames ac turpis egestas. Mauris ut leo. Cras viverra metus rhoncus sem. Nulla et lectus vestibulum urna fringilla ultrices. Phasellus eu tellus sit amet tortor gravida placerat. Integer sapien est, iaculis in, pretium quis, viverra ac, nunc. Praesent eget sem vel leo ultrices bibendum. Aenean faucibus. Morbi dolor nulla, malesuada eu, pulvinar at, mollis ac, nulla. Curabitur auctor semper nulla. Donec varius orci eget risus. Duis nibh mi, congue eu, accumsan eleifend, sagittis quis, diam. Duis eget orci sit amet orci dignissim rutrum.

PTSansCaption-TLF PT Sans Caption

Lorem ipsum dolor sit amet, consectetuer adipiscing elit. Ut purus elit, vestibulum ut, placerat ac, adipiscing vitae, felis. Curabitur dictum gravida mauris. Nam arcu libero, nonummy eget, consectetuer id, vulputate a, magna. Donec vehicula augue eu neque. Pellentesque habitant morbi tristique senectus et netus et malesuada fames ac turpis egestas. Mauris ut leo. Cras viverra metus rhoncus sem. Nulla et lectus vestibulum urna fringilla ultrices. Phasellus eu tellus sit amet tortor gravida placerat. Integer sapien est, iaculis in, pretium quis, viverra ac, nunc. Praesent eget sem vel leo ultrices bibendum. Aenean faucibus. Morbi dolor nulla, malesuada eu, pulvinar at, mollis ac, nulla. Curabitur auctor semper nulla. Donec varius orci eget risus. Duis nibh mi, congue eu, accumsan eleifend, sagittis quis, diam. Duis eget orci sit amet orci dignissim rutrum.

DejaVuSans-TLF DejaVuSans

DejaVuSans-TLF DejaVuSans

DejaVu Sans	**DejaVu Sans Bold**
DejaVu Sans Oblique	***DejaVu Sans Bold Oblique***
DejaVu Sans Condensed	**DejaVu Sans Condensed Bold**
DejaVu Sans Condensed Oblique	***DejaVu Sans Condensed Bold Oblique***
DejaVu Sans ExtraLight	

Additional resources

The LaTeX Font Catalogue at `http://www.tug.dk/FontCatalogue/` is a start.

Chapter 4

Working with math

4.1 AMS Introduction

This paper contains examples of various features from $\mathcal{A}\mathcal{M}\mathcal{S}$-LaTeX and was abstracted from the file `testmath.tex`. The citations have been removed.

4.2 Enumeration of Hamiltonian Paths in a Graph

Let $\mathbf{A} = (a_{ij})$ be the adjacency matrix of graph G. The corresponding Kirchhoff matrix $\mathbf{K} = (k_{ij})$ is obtained from $\mathbf{A}$ by replacing in $-\mathbf{A}$ each diagonal entry by the degree of its corresponding vertex; i.e., the ith diagonal entry is identified with the degree of the ith vertex. It is well known that

$$\det\mathbf{K}(i|i) = \text{ the number of spanning trees of } G, \quad i = 1, \dots, n \tag{4.1}$$

where $\mathbf{K}(i|i)$ is the ith principal submatrix of $\mathbf{K}$.

```
\det\mathbf{K}(i|i)=\text{ the number of spanning trees of $G$},
```

Let $C_{i(j)}$ be the set of graphs obtained from G by attaching edge $(v_i v_j)$ to each spanning tree of G. Denote by $C_i = \bigcup_j C_{i(j)}$. It is obvious that the collection of Hamiltonian cycles is a subset of C_i. Note that the cardinality of C_i is $k_{ii}\det\mathbf{K}(i|i)$. Let $X = \{\hat x_1, \dots, \hat x_n\}$.

```
$X=\{\hat x_1,\dots,\hat x_n\}$
```

Define multiplication for the elements of X by

$$\hat x_i \hat x_j = \hat x_j \hat x_i, \quad \hat x_i^2 = 0, \quad i, j = 1, \dots, n. \tag{4.2}$$

Let $\hat k_{ij} = k_{ij}\hat x_j$ and $\hat k_{ij} = -\sum_{j\neq i}\hat k_{ij}$. Then the number of Hamiltonian cycles H_c is given by the relation

$$\left(\prod_{j=1}^{n}\hat x_j\right)H_c = \frac{1}{2}\hat k_{ij}\det\mathbf{K}(i|i), \qquad i = 1, \dots, n. \tag{4.3}$$

The task here is to express (4.3) in a form free of any $\hat x_i$, $i = 1, \dots, n$. The result also leads to the resolution of enumeration of Hamiltonian paths in a graph.

It is well known that the enumeration of Hamiltonian cycles and paths in a complete graph K_n and in a complete bipartite graph $K_{n_1 n_2}$ can only be found from *first combinatorial principles*. One wonders if there exists a formula which can be used very efficiently to produce K_n and $K_{n_1 n_2}$. Recently, using Lagrangian methods, Goulden and Jackson have shown that H_c can be expressed in terms of the determinant and permanent of the adjacency matrix. However, the formula of Goulden and Jackson determines neither K_n nor $K_{n_1 n_2}$ effectively. In this paper, using an algebraic method, we parametrize the adjacency matrix. The resulting formula also involves the determinant and permanent, but it can easily be applied to K_n and $K_{n_1 n_2}$. In addition, we eliminate the permanent from H_c and show that H_c can be represented by a determinantal function of multivariables, each variable with domain $\{0,1\}$. Furthermore, we show that H_c can be written by number of spanning trees of subgraphs. Finally, we apply the formulas to a complete multigraph $K_{n_1 \dots n_p}$.

The conditions $a_{ij} = a_{ji}$, $i, j = 1, \dots, n$, are not required in this paper. All formulas can be extended to a digraph simply by multiplying H_c by 2.

4.3 Application

We consider here the applications of these Theorems to a complete multipartite graph $K_{n_1 \dots n_p}$. It can be shown that the number of spanning trees of $K_{n_1 \dots n_p}$ may be written

$$T = n^{p-2} \prod_{i=1}^{p} (n - n_i)^{n_i - 1} \tag{4.4}$$

where

$$n = n_1 + \cdots + n_p. \tag{4.5}$$

It follows that

$$\begin{aligned} H_c = \frac{1}{2n} \sum_{l=0}^{n} (-1)^l (n-l)^{p-2} \sum_{l_1 + \cdots + l_p = l} \prod_{i=1}^{p} \binom{n_i}{l_i} \\ \cdot [(n-l) - (n_i - l_i)]^{n_i - l_i} \cdot \left[(n-l)^2 - \sum_{j=1}^{p} (n_i - l_i)^2 \right]. \end{aligned} \tag{4.6}$$

```
... \binom{n_i}{l _i}\\
```

and

$$\begin{aligned} H_c = \frac{1}{2} \sum_{l=0}^{n-1} (-1)^l (n-l)^{p-2} \sum_{l_1 + \cdots + l_p = l} \prod_{i=1}^{p} \binom{n_i}{l_i} \\ \cdot [(n-l) - (n_i - l_i)]^{n_i - l_i} \left(1 - \frac{l_p}{n_p} \right) [(n-l) - (n_p - l_p)]. \end{aligned} \tag{4.7}$$

The enumeration of H_c in a $K_{n_1\cdots n_p}$ graph can also be carried out by these theorems together with the algebraic method of (4.2). Some elegant representations may be obtained. For example, H_c in a $K_{n_1 n_2 n_3}$ graph may be written

$$\begin{aligned} H_c =& \frac{n_1!n_2!n_3!}{n_1+n_2+n_3} \sum_i \left[\binom{n_1}{i} \binom{n_2}{n_3-n_1+i} \binom{n_3}{n_3-n_2+i} \right. \\ & \left. + \binom{n_1-1}{i} \binom{n_2-1}{n_3-n_1+i} \binom{n_3-1}{n_3-n_2+i} \right]. \end{aligned} \tag{4.8}$$

4.4 Secret Key Exchanges

Modern cryptography is fundamentally concerned with the problem of secure private communication. A Secret Key Exchange is a protocol where Alice and Bob, having no secret information in common to start, are able to agree on a common secret key, conversing over a public channel. The notion of a Secret Key Exchange protocol was first introduced in the seminal paper of Diffie and Hellman which presented a concrete implementation of a Secret Key Exchange protocol, dependent on a specific assumption (a variant on the discrete log), specially tailored to yield Secret Key Exchange. Secret Key Exchange is of course trivial if trapdoor permutations exist. However, there is no known implementation based on a weaker general assumption.

Chapter 5

Results

5.1 Super results

Nulla malesuada porttitor diam. Donec felis erat, congue non, volutpat at, tincidunt tristique, libero. Vivamus viverra fermentum felis. Donec nonummy pellentesque ante. Phasellus adipiscing semper elit. Proin fermentum massa ac quam. Sed diam turpis, molestie vitae, placerat a, molestie nec, leo. Maecenas lacinia. Nam ipsum ligula, eleifend at, accumsan nec, suscipit a, ipsum. Morbi blandit ligula feugiat magna. Nunc eleifend consequat lorem. Sed lacinia nulla vitae enim. Pellentesque tincidunt purus vel magna. Integer non enim. Praesent euismod nunc eu purus. Donec bibendum quam in tellus. Nullam cursus pulvinar lectus. Donec et mi. Nam vulputate metus eu enim. Vestibulum pellentesque felis eu massa.

5.2 Using figures

We demonstrate a technique to set up and configure a figure using the `\includegraphics` command embedded into the picture environment embedded into the figure environment. The picture environment is needed to control the spacing, while the figure environment is needed to control the floating of the figures in the document. Note that you can refer to the figure with its label, i.e., in Figure 5.2 you have the setup of the figure. Note that you need to create the figures so that the labels and legends scale well with the fonts being used after you have scaled the figure. The examples in Fig. 5.1 and Fig. 5.2 should be examined in detail as the axes labels are not generated with the figures. Instead, they are added as overlays using the `\put` command in the `picture` environment.

Pellentesque habitant morbi tristique senectus et netus et malesuada fames ac turpis egestas. Donec odio elit, dictum in, hendrerit sit amet, egestas sed, leo. Praesent feugiat sapien aliquet odio. Integer vitae justo. Aliquam vestibulum fringilla lorem. Sed neque lectus, consectetuer at, consectetuer sed, eleifend ac, lectus. Nulla facilisi. Pellentesque eget lectus. Proin eu metus. Sed porttitor. In hac habitasse platea dictumst. Suspendisse eu lectus. Ut mi mi, lacinia sit amet, placerat et, mollis vitae, dui. Sed ante tellus, tristique ut, iaculis eu, malesuada ac, dui. Mauris nibh leo, facilisis non, adipiscing quis, ultrices a, dui.

Morbi luctus, wisi viverra faucibus pretium, nibh est placerat odio, nec commodo wisi enim eget quam. Quisque libero justo, consectetuer a, feugiat vitae, porttitor eu, libero.

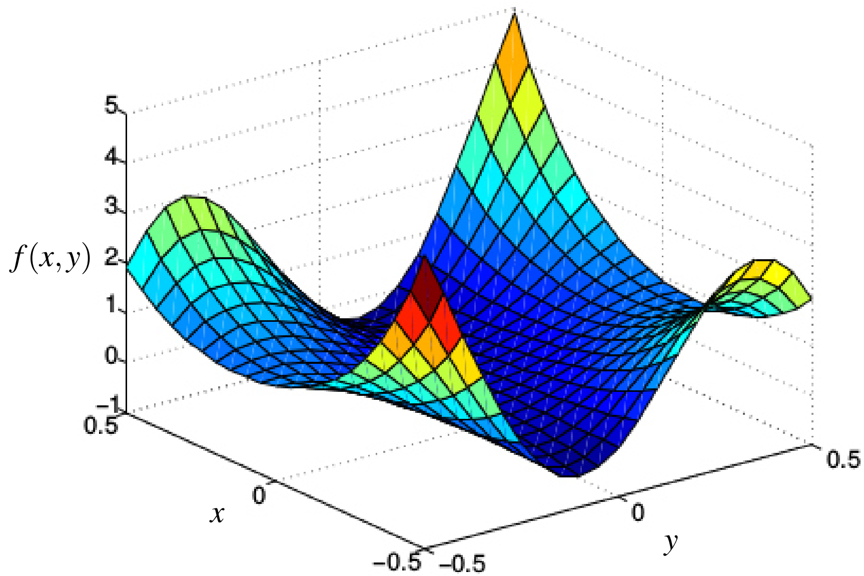

Figure 5.1: Example of using a raster image which is scaled and annotated with axes.

Suspendisse sed mauris vitae elit sollicitudin malesuada. Maecenas ultricies eros sit amet ante. Ut venenatis velit. Maecenas sed mi eget dui varius euismod. Phasellus aliquet volutpat odio. Vestibulum ante ipsum primis in faucibus orci luctus et ultrices posuere cubilia Curae; Pellentesque sit amet pede ac sem eleifend consectetuer. Nullam elementum, urna vel imperdiet sodales, elit ipsum pharetra ligula, ac pretium ante justo a nulla. Curabitur tristique arcu eu metus. Vestibulum lectus. Proin mauris. Proin eu nunc eu urna hendrerit faucibus. Aliquam auctor, pede consequat laoreet varius, eros tellus scelerisque quam, pellentesque hendrerit ipsum dolor sed augue. Nulla nec lacus.

Suspendisse vitae elit. Aliquam arcu neque, ornare in, ullamcorper quis, commodo eu, libero. Fusce sagittis erat at erat tristique mollis. Maecenas sapien libero, molestie et, lobortis in, sodales eget, dui. Morbi ultrices rutrum lorem. Nam elementum ullamcorper leo. Morbi dui. Aliquam sagittis. Nunc placerat. Pellentesque tristique sodales est. Maecenas imperdiet lacinia velit. Cras non urna. Morbi eros pede, suscipit ac, varius vel, egestas non, eros. Praesent malesuada, diam id pretium elementum, eros sem dictum tortor, vel consectetuer odio sem sed wisi.

Sed feugiat. Cum sociis natoque penatibus et magnis dis parturient montes, nascetur ridiculus mus. Ut pellentesque augue sed urna. Vestibulum diam eros, fringilla et, consectetuer eu, nonummy id, sapien. Nullam at lectus. In sagittis ultrices mauris. Curabitur malesuada erat sit amet massa. Fusce blandit. Aliquam erat volutpat. Aliquam euismod. Aenean vel lectus. Nunc imperdiet justo nec dolor.

Etiam euismod. Fusce facilisis lacinia dui. Suspendisse potenti. In mi erat, cursus id, nonummy sed, ullamcorper eget, sapien. Praesent pretium, magna in eleifend egestas, pede pede pretium lorem, quis consectetuer tortor sapien facilisis magna. Mauris quis magna varius nulla scelerisque imperdiet. Aliquam non quam. Aliquam porttitor quam a lacus. Praesent vel arcu ut tortor cursus volutpat. In vitae pede quis diam bibendum placerat. Fusce elementum convallis neque. Sed dolor orci, scelerisque ac, dapibus nec, ultricies ut, mi. Duis nec dui quis leo sagittis commodo.

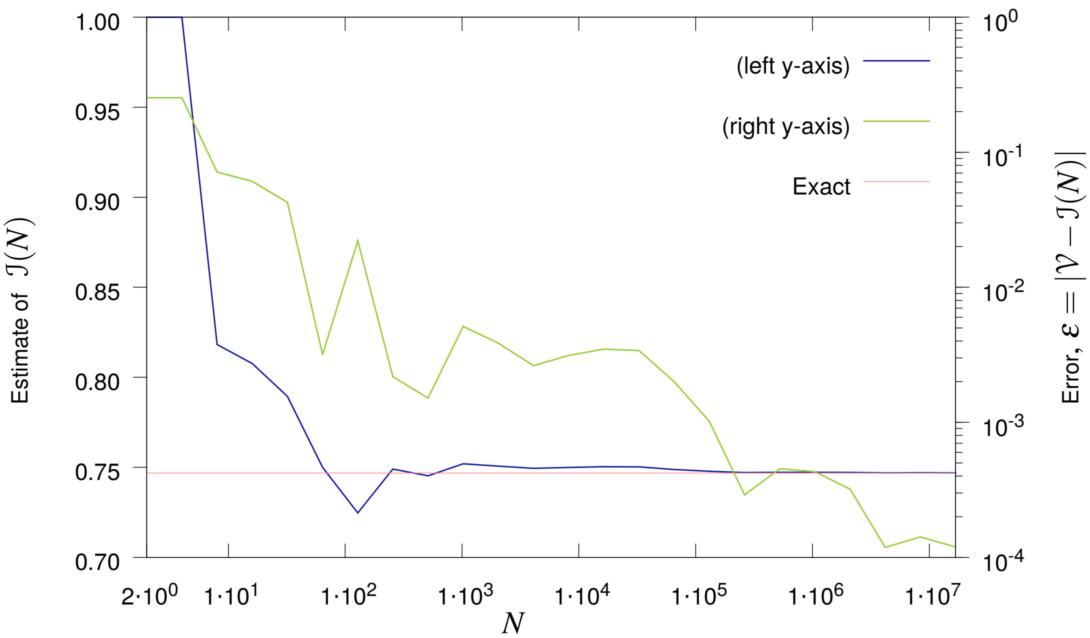

Figure 5.2: An approximation $\mathcal{I}(N)$ to the value of the integral for $\int_0^1 e^{x^2} dx$ obtained using a Monte Carlo approach. The estimate for $\mathcal{I}(N)$ is a function of the number of samples N, and the value of $I \approx 0.7468241328 \equiv \mathcal{V}$ was obtained using Wolfram Alpha®.

5.3 Using another figure

Nulla malesuada porttitor diam. Donec felis erat, congue non, volutpat at, tincidunt tristique, libero. Vivamus viverra fermentum felis. Donec nonummy pellentesque ante. Phasellus adipiscing semper elit. Proin fermentum massa ac quam. Sed diam turpis, molestie vitae, placerat a, molestie nec, leo. Maecenas lacinia. Nam ipsum ligula, eleifend at, accumsan nec, suscipit a, ipsum. Morbi blandit ligula feugiat magna. Nunc eleifend consequat lorem. Sed lacinia nulla vitae enim. Pellentesque tincidunt purus vel magna. Integer non enim. Praesent euismod nunc eu purus. Donec bibendum quam in tellus. Nullam cursus pulvinar lectus. Donec et mi. Nam vulputate metus eu enim. Vestibulum pellentesque felis eu massa.

Quisque ullamcorper placerat ipsum. Cras nibh. Morbi vel justo vitae lacus tincidunt ultrices. Lorem ipsum dolor sit amet, consectetuer adipiscing elit. In hac habitasse platea dictumst. Integer tempus convallis augue. Etiam facilisis. Nunc elementum fermentum wisi. Aenean placerat. Ut imperdiet, enim sed gravida sollicitudin, felis odio placerat quam, ac pulvinar elit purus eget enim. Nunc vitae tortor. Proin tempus nibh sit amet nisl. Vivamus quis tortor vitae risus porta vehicula.

Fusce mauris. Vestibulum luctus nibh at lectus. Sed bibendum, nulla a faucibus semper, leo velit ultricies tellus, ac venenatis arcu wisi vel nisl. Vestibulum diam. Aliquam pellentesque, augue quis sagittis posuere, turpis lacus congue quam, in hendrerit risus eros eget felis. Maecenas eget erat in sapien mattis porttitor. Vestibulum porttitor. Nulla facilisi. Sed a turpis eu lacus commodo facilisis. Morbi fringilla, wisi in dignissim interdum, justo lectus sagittis dui, et vehicula libero dui cursus dui. Mauris tempor ligula sed lacus. Duis cursus enim ut augue. Cras ac magna. Cras nulla. Nulla egestas. Curabitur a leo. Quisque egestas wisi eget nunc. Nam feugiat lacus vel est. Curabitur consectetuer.

i	1	2	3	4	5	6	7
P_i	0.34	0.11	0.11	0.11	0.11	0.11	0.11
$l(P)$	1	2	2	2	2	2	2
code(P)	0	10	11	12	20	21	22
Ideal Length	0.982	2.009	2.009	2.009	2.009	2.009	2.009

Table 5.1: Tabulated results in base 3 with actual and ideal code length.

Nulla malesuada porttitor diam. Donec felis erat, congue non, volutpat at, tincidunt tristique, libero. Vivamus viverra fermentum felis. Donec nonummy pellentesque ante. Phasellus adipiscing semper elit. Proin fermentum massa ac quam. Sed diam turpis, molestie vitae, placerat a, molestie nec, leo. Maecenas lacinia. Nam ipsum ligula, eleifend at, accumsan nec, suscipit a, ipsum. Morbi blandit ligula feugiat magna. Nunc eleifend consequat lorem. Sed lacinia nulla vitae enim. Pellentesque tincidunt purus vel magna. Integer non enim. Praesent euismod nunc eu purus. Donec bibendum quam in tellus. Nullam cursus pulvinar lectus. Donec et mi. Nam vulputate metus eu enim. Vestibulum pellentesque felis eu massa.

An alternative style of table is present in which the fonts are fixed spacing and the rows are alternately colored to improve readability.

Nulla malesuada porttitor diam. Donec felis erat, congue non, volutpat at, tincidunt tristique, libero. Vivamus viverra fermentum felis. Donec nonummy pellentesque ante. Phasellus adipiscing semper elit. Proin fermentum massa ac quam. Sed diam turpis, molestie vitae, placerat a, molestie nec, leo. Maecenas lacinia. Nam ipsum ligula, eleifend at, accumsan nec, suscipit a, ipsum. Morbi blandit ligula feugiat magna. Nunc eleifend consequat lorem. Sed lacinia nulla vitae enim. Pellentesque tincidunt purus vel magna. Integer non enim. Praesent euismod nunc eu purus. Donec bibendum quam in tellus. Nullam

N	$\mathcal{I}(N)$	Error, ε
2	1.00000000	0.25317587
4	1.00000000	0.25317587
8	0.81818182	0.07135769
16	0.80769231	0.06086817
32	0.78947368	0.04264955
64	0.75000000	0.00317587
128	0.72469636	0.02212778
256	0.74900398	0.00217985
512	0.74531096	0.00151318
1024	0.75196464	0.00514050
2048	0.75067352	0.00384939
4096	0.74944974	0.00262561
8192	0.74995418	0.00313005
16384	0.75030532	0.00348119
32768	0.75022513	0.00340099
65536	0.74882110	0.00199696
131072	0.74783786	0.00101373
262144	0.74711407	0.00028994
524288	0.74727697	0.00045283
1048576	0.74725506	0.00043093
2097152	0.74714403	0.00031989
4194304	0.74694263	0.00011850
8388608	0.74696581	0.00014167
16777216	0.74694349	0.00011935

Table 5.3: Data generated using the code, `monte.py`. The output in this table is generated by `monte-latex.py` which has LaTeXformatting included. This table illustrates the use of color, and while color is typically not used in most journal publications, it does provide a means of making reports and other online articles more readable. In this example the use of a custom font is also illustrated.

cursus pulvinar lectus. Donec et mi. Nam vulputate metus eu enim. Vestibulum pellentesque felis eu massa.

Quisque ullamcorper placerat ipsum. Cras nibh. Morbi vel justo vitae lacus tincidunt ultrices. Lorem ipsum dolor sit amet, consectetuer adipiscing elit. In hac habitasse platea dictumst. Integer tempus convallis augue. Etiam facilisis. Nunc elementum fermentum wisi. Aenean placerat. Ut imperdiet, enim sed gravida sollicitudin, felis odio placerat quam, ac pulvinar elit purus eget enim. Nunc vitae tortor. Proin tempus nibh sit amet nisl. Vivamus quis tortor vitae risus porta vehicula.

Fusce mauris. Vestibulum luctus nibh at lectus. Sed bibendum, nulla a faucibus semper, leo velit ultricies tellus, ac venenatis arcu wisi vel nisl. Vestibulum diam. Aliquam pellentesque, augue quis sagittis posuere, turpis lacus congue quam, in hendrerit risus eros eget felis. Maecenas eget erat in sapien mattis porttitor. Vestibulum porttitor. Nulla facilisi. Sed a turpis eu lacus commodo facilisis. Morbi fringilla, wisi in dignissim interdum, justo lectus sagittis dui, et vehicula libero dui cursus dui. Mauris tempor ligula sed lacus. Duis cursus enim ut augue. Cras ac magna. Cras nulla. Nulla egestas. Curabitur a leo. Quisque egestas wisi eget nunc. Nam feugiat lacus vel est. Curabitur consectetuer.

Suspendisse vel felis. Ut lorem lorem, interdum eu, tincidunt sit amet, laoreet vitae, arcu. Aenean faucibus pede eu ante. Praesent enim elit, rutrum at, molestie non, nonummy vel, nisl. Ut lectus eros, malesuada sit amet, fermentum eu, sodales cursus, magna. Donec eu purus. Quisque vehicula, urna sed ultricies auctor, pede lorem egestas dui, et convallis elit erat sed nulla. Donec luctus. Curabitur et nunc. Aliquam dolor odio, commodo pretium, ultricies non, pharetra in, velit. Integer arcu est, nonummy in, fermentum faucibus, egestas vel, odio.

Sed commodo posuere pede. Mauris ut est. Ut quis purus. Sed ac odio. Sed vehicula hendrerit sem. Duis non odio. Morbi ut dui. Sed accumsan risus eget odio. In hac habitasse platea dictumst. Pellentesque non elit. Fusce sed justo eu urna porta tincidunt. Mauris felis odio, sollicitudin sed, volutpat a, ornare ac, erat. Morbi quis dolor. Donec pellentesque, erat ac sagittis semper, nunc dui lobortis purus, quis congue purus metus ultricies tellus. Proin et quam. Class aptent taciti sociosqu ad litora torquent per conubia nostra, per inceptos hymenaeos. Praesent sapien turpis, fermentum vel, eleifend faucibus, vehicula eu, lacus.

5.4 Even more results

Morbi luctus, wisi viverra faucibus pretium, nibh est placerat odio, nec commodo wisi enim eget quam. Quisque libero justo, consectetuer a, feugiat vitae, porttitor eu, libero. Suspendisse sed mauris vitae elit sollicitudin malesuada. Maecenas ultricies eros sit amet ante. Ut venenatis velit. Maecenas sed mi eget dui varius euismod. Phasellus aliquet volutpat odio. Vestibulum ante ipsum primis in faucibus orci luctus et ultrices posuere cubilia Curae; Pellentesque sit amet pede ac sem eleifend consectetuer. Nullam elementum, urna vel imperdiet sodales, elit ipsum pharetra ligula, ac pretium ante justo a nulla. Curabitur tristique arcu eu metus. Vestibulum lectus. Proin mauris. Proin eu nunc eu urna hendrerit faucibus. Aliquam auctor, pede consequat laoreet varius, eros tellus scelerisque quam, pellentesque hendrerit ipsum dolor sed augue. Nulla nec lacus.

Suspendisse vitae elit. Aliquam arcu neque, ornare in, ullamcorper quis, commodo eu, libero. Fusce sagittis erat at erat tristique mollis. Maecenas sapien libero, molestie et, lobortis in, sodales eget, dui. Morbi ultrices rutrum lorem. Nam elementum ullamcorper leo. Morbi dui. Aliquam sagittis. Nunc placerat. Pellentesque tristique sodales est. Maecenas imperdiet lacinia velit. Cras non urna. Morbi eros pede, suscipit ac, varius vel, egestas non, eros. Praesent malesuada, diam id pretium elementum, eros sem dictum tortor, vel consectetuer odio sem sed wisi.

Sed feugiat. Cum sociis natoque penatibus et magnis dis parturient montes, nascetur ridiculus mus. Ut pellentesque augue sed urna. Vestibulum diam eros, fringilla et, consectetuer eu, nonummy id, sapien. Nullam at lectus. In sagittis ultrices mauris. Curabitur malesuada erat sit amet massa. Fusce blandit. Aliquam erat volutpat. Aliquam euismod. Aenean vel lectus. Nunc imperdiet justo nec dolor.

Etiam euismod. Fusce facilisis lacinia dui. Suspendisse potenti. In mi erat, cursus id, nonummy sed, ullamcorper eget, sapien. Praesent pretium, magna in eleifend egestas, pede pede pretium lorem, quis consectetuer tortor sapien facilisis magna. Mauris quis magna varius nulla scelerisque imperdiet. Aliquam non quam. Aliquam porttitor quam a lacus. Praesent vel arcu ut tortor cursus volutpat. In vitae pede quis diam bibendum placerat. Fusce elementum convallis neque. Sed dolor orci, scelerisque ac, dapibus nec, ultricies ut, mi. Duis nec dui quis leo sagittis commodo.

Chapter 6

Conclusions and Future Work

6.1 Known Results

Everything we have done is completely correct based on the theory of everything. Etiam euismod. Fusce facilisis lacinia dui. Suspendisse potenti. In mi erat, cursus id, nonummy sed, ullamcorper eget, sapien. Praesent pretium, magna in eleifend egestas, pede pede pretium lorem, quis consectetuer tortor sapien facilisis magna. Mauris quis magna varius nulla scelerisque imperdiet. Aliquam non quam. Aliquam porttitor quam a lacus. Praesent vel arcu ut tortor cursus volutpat. In vitae pede quis diam bibendum placerat. Fusce elementum convallis neque. Sed dolor orci, scelerisque ac, dapibus nec, ultricies ut, mi. Duis nec dui quis leo sagittis commodo.

Aliquam lectus. Vivamus leo. Quisque ornare tellus ullamcorper nulla. Mauris porttitor pharetra tortor. Sed fringilla justo sed mauris. Mauris tellus. Sed non leo. Nullam elementum, magna in cursus sodales, augue est scelerisque sapien, venenatis congue nulla arcu et pede. Ut suscipit enim vel sapien. Donec congue. Maecenas urna mi, suscipit in, placerat ut, vestibulum ut, massa. Fusce ultrices nulla et nisl.

Etiam ac leo a risus tristique nonummy. Donec dignissim tincidunt nulla. Vestibulum rhoncus molestie odio. Sed lobortis, justo et pretium lobortis, mauris turpis condimentum augue, nec ultricies nibh arcu pretium enim. Nunc purus neque, placerat id, imperdiet sed, pellentesque nec, nisl. Vestibulum imperdiet neque non sem accumsan laoreet. In hac habitasse platea dictumst. Etiam condimentum facilisis libero. Suspendisse in elit quis nisl aliquam dapibus. Pellentesque auctor sapien. Sed egestas sapien nec lectus. Pellentesque vel dui vel neque bibendum viverra. Aliquam porttitor nisl nec pede. Proin mattis libero vel turpis. Donec rutrum mauris et libero. Proin euismod porta felis. Nam lobortis, metus quis elementum commodo, nunc lectus elementum mauris, eget vulputate ligula tellus eu neque. Vivamus eu dolor.

Nulla in ipsum. Praesent eros nulla, congue vitae, euismod ut, commodo a, wisi. Pellentesque habitant morbi tristique senectus et netus et malesuada fames ac turpis egestas. Aenean nonummy magna non leo. Sed felis erat, ullamcorper in, dictum non, ultricies ut, lectus. Proin vel arcu a odio lobortis euismod. Vestibulum ante ipsum primis in faucibus orci luctus et ultrices posuere cubilia Curae; Proin ut est. Aliquam odio. Pellentesque massa turpis, cursus eu, euismod nec, tempor congue, nulla. Duis viverra gravida mauris. Cras tincidunt. Curabitur eros ligula, varius ut, pulvinar in, cursus faucibus, augue.

Nulla mattis luctus nulla. Duis commodo velit at leo. Aliquam vulputate magna et leo. Nam vestibulum ullamcorper leo. Vestibulum condimentum rutrum mauris. Donec id mauris. Morbi molestie justo et pede. Vivamus eget turpis sed nisl cursus tempor. Curabitur mollis sapien condimentum nunc. In wisi nisl, malesuada at, dignissim sit amet, lobortis in, odio. Aenean consequat arcu a ante. Pellentesque porta elit sit amet orci. Etiam at turpis nec elit ultricies imperdiet. Nulla facilisi. In hac habitasse platea dictumst. Suspendisse viverra aliquam risus. Nullam pede justo, molestie nonummy, scelerisque eu, facilisis vel, arcu.

Curabitur tellus magna, porttitor a, commodo a, commodo in, tortor. Donec interdum. Praesent scelerisque. Maecenas posuere sodales odio. Vivamus metus lacus, varius quis, imperdiet quis, rhoncus a, turpis. Etiam ligula arcu, elementum a, venenatis quis, sollicitudin sed, metus. Donec nunc pede, tincidunt in, venenatis vitae, faucibus vel, nibh. Pellentesque wisi. Nullam malesuada. Morbi ut tellus ut pede tincidunt porta. Lorem ipsum dolor sit amet, consectetuer adipiscing elit. Etiam congue neque id dolor.

Donec et nisl at wisi luctus bibendum. Nam interdum tellus ac libero. Sed sem justo, laoreet vitae, fringilla at, adipiscing ut, nibh. Maecenas non sem quis tortor eleifend fermentum. Etiam id tortor ac mauris porta vulputate. Integer porta neque vitae massa. Maecenas tempus libero a libero posuere dictum. Vestibulum ante ipsum primis in faucibus orci luctus et ultrices posuere cubilia Curae; Aenean quis mauris sed elit commodo placerat. Class aptent taciti sociosqu ad litora torquent per conubia nostra, per inceptos hymenaeos. Vivamus rhoncus tincidunt libero. Etiam elementum pretium justo. Vivamus est. Morbi a tellus eget pede tristique commodo. Nulla nisl. Vestibulum sed nisl eu sapien cursus rutrum.

Nulla non mauris vitae wisi posuere convallis. Sed eu nulla nec eros scelerisque pharetra. Nullam varius. Etiam dignissim elementum metus. Vestibulum faucibus, metus sit amet mattis rhoncus, sapien dui laoreet odio, nec ultricies nibh augue a enim. Fusce in ligula. Quisque at magna et nulla commodo consequat. Proin accumsan imperdiet sem. Nunc porta. Donec feugiat mi at justo. Phasellus facilisis ipsum quis ante. In ac elit eget ipsum pharetra faucibus. Maecenas viverra nulla in massa.

Nulla ac nisl. Nullam urna nulla, ullamcorper in, interdum sit amet, gravida ut, risus. Aenean ac enim. In luctus. Phasellus eu quam vitae turpis viverra pellentesque. Duis feugiat felis ut enim. Phasellus pharetra, sem id porttitor sodales, magna nunc aliquet nibh, nec blandit nisl mauris at pede. Suspendisse risus risus, lobortis eget, semper at, imperdiet sit amet, quam. Quisque scelerisque dapibus nibh. Nam enim. Lorem ipsum dolor sit amet, consectetuer adipiscing elit. Nunc ut metus. Ut metus justo, auctor at, ultrices eu, sagittis ut, purus. Aliquam aliquam.

6.2 Wrapup

Where can we find information about this thesis style? Go to http://math.newhaven.edu/mathphysics.

Appendices

Appendix A

Software listings

A.1 Code listing

In Appendices A, B, C, D we examine several methods for including software listings into LaTeX . The method used in this appendix is the most complex approach using the python script, pygmentize, which allows for a full range of options, however the resulting text that is generated is not the actual code, but mostly markups of the code that make it very difficult to read from the LaTeX source itself. Compare the source files, `chap-appendixa.tex` and `chap-appendixb.tex` for the code `monte.py` to view the difference. The advantage to using pymgmentize which is part of the Pygments software package at http://pygments.org/ is that it is a cross-platform python based syntax highlighter that can produce output in any of several output formats, including LaTeX and HTML, and it can work with an enormously wide range of input formats.

```
#!/usr/bin/python
from random import *
from math import sqrt, exp
fo=open('data.dat','w')

INT=0.7468241328
M=25
N=1
hits = 0
throws = 0
seed(2303417331)
for j in range (1, M):
        N=2*N
        for i in range (1, N):
                throws += 1
                x = random()
                y = random()
                f = exp(-x*x)
                if y <= f:
                        hits = hits + 1.0
        area = (hits / throws)
        error = abs(area - INT)
        print  ("{0:8d} {1:.8f} {2:.8f}".format(N, area, error))
        print >> fo, ("{0:8d} {1:.8f} {2:.8f}".format(N, area, error))

fo.close
```

Appendix B

More software listings

B.1 Another Code listing

This is the most foundational method, consisting of a verbatim listing of the code wrappered around a coloring of the text to make it stand out from the other text on the page.

Code listing for `monte.py`

```
#!/usr/bin/python
from random import *
from math import sqrt, exp
fo=open('data.dat','w')

INT=0.7468241328
M=25
N=1
hits = 0
throws = 0
seed(2303417331)
for j in range (1, M):
N=2*N
for i in range (1, N):
throws += 1
x = random()
y = random()
f = exp(-x*x)
if y <= f:
hits = hits + 1.0
area = (hits / throws)
        error = abs(area - INT)
print  ("{0:8d} {1:.8f} {2:.8f}".format(N, area, error))
print >> fo, ("{0:8d} {1:.8f} {2:.8f}".format(N, area, error))

fo.close
```

Code listing for `monte-latex.py`

```
#!/usr/bin/python
from random import *
from math import sqrt, exp
ft=open('data.tex','w')
fo=open('data.dat','w')

INT=0.7468241328
M=25
N=1
hits = 0
throws = 0
seed(73417331)
print >> ft, ("{\\usefont{T1}{jkptt}{m}{n}\\fontsize{9.0}{10.8}\\selectfont")
print >> ft, ("\\begin{longtable}{|r|r|r|}")
print >> ft, ("\\hline")
```

```
print >> ft, ("$N$ & ${\cal I}(N)$ \
 \\hspace*{0.8em} & \\textbf{\\Hf Error}, $\\varepsilon$ \\hspace*{0.2em} \\\\")
print >> ft, ("\\hline")
for j in range (1, M):
N=2*N
for i in range (1, N):
throws += 1
x = random()
y = random()
f = exp(-x*x)
if y <= f:
hits = hits + 1.0
area = (hits / throws)
        error = abs(area - INT)
print  ("{0:8d} {1:.8f} {2:.8f}".format(N, area, error))
print >> fo, ("{0:8d} {1:.8f} {2:.8f}".format(N, area, error))
print >> ft, ("{0:8d} & {1:.8f} & {2:.8f} \\\\".format(N, area, error))
# f.write("{0:8d} {1:.8f} {2:.8f}" % (N, area, error))

print >> ft, ("\\hline")
print >> ft, ("\\end{longtable}}")
fo.close
ft.close
```

Appendix C

Another approach to software listings

C.1 Still another way of listing

This method uses the `listings` package to provide a means of listing software. The advantage over using pygmentize is that the code is actually readable, while the text generated by pygmentize is not readable. Also, packages usually have quite a range of options that can be customized.

```java
public class Hello
{
    public static void main(String[] args)
    {
        // print hello to the console
        System.out.println("Hello, world!");
    }
}
```

Listing C.1: Java code using listings.

```python
from random import random
from math import sqrt, exp

INT=0.7468241328
M=25
N=1
hits = 0
throws = 0
for j in range (1, M):
        N=2*N
        for i in range (1, N):
                throws += 1
                x = random()
                y = random()
                f = exp(-x*x)
                if y <= f:
                        hits = hits + 1.0
        area = (hits / throws)
        error = abs(area - INT)
#        print " area = %s" %(area)
        print ("{0:8d} {1:.8f} {2:.8f}".format(N, area, error))
```

Listing C.2: Python code using listings.

```cpp
#include <iostream>
int main()
{
    // print hello to the console
    std::cout << "Hello, world!" << std::endl;
    return 0;
}
```

Listing C.3: C++ code using listings.

Appendix D

Color names

D.1 The `xcolor` color names

The next few pages lists all of the colors provided when the color options used by the `xcolor` LaTeX package. This list is taken from the `xcolor` package documentation and is intended to be used as a reference for selecting a color to use within templates.

It is also worth noting that the pages that are included in this appendix are pulled in from a pdf file using the command,

```
\includepdf[pages={1-3}, nup={1x1},scale={0.9},delta={0.0pt 0.0pt}]
   {figures/xcolors.pdf}
```

This is an essential and valuable package, however it breaks the `hyperref` package, and so links are not possible on these pages.

4 Colors by Name

4.1 Base colors (always available)

black
blue
brown
cyan
darkgray
gray
green
lightgray
lime
magenta
olive
orange
pink
purple
red
teal
violet
white
yellow

4.2 Colors via `dvipsnames` option

Apricot
Aquamarine
Bittersweet
Black
Blue
BlueGreen
BlueViolet
BrickRed
Brown
BurntOrange
CadetBlue
CarnationPink
Cerulean
CornflowerBlue
Cyan
Dandelion
DarkOrchid
Emerald
ForestGreen
Fuchsia
Goldenrod
Gray
Green
GreenYellow
JungleGreen
Lavender
LimeGreen
Magenta
Mahogany
Maroon
Melon
MidnightBlue
Mulberry
NavyBlue
OliveGreen
Orange
OrangeRed
Orchid
Peach
Periwinkle
PineGreen
Plum
ProcessBlue
Purple
RawSienna
Red
RedOrange
RedViolet
Rhodamine
RoyalBlue
RoyalPurple
RubineRed
Salmon
SeaGreen
Sepia
SkyBlue
SpringGreen
Tan
TealBlue
Thistle
Turquoise
Violet
VioletRed
White
WildStrawberry
Yellow
YellowGreen
YellowOrange

4.3 Colors via `svgnames` option

AliceBlue
AntiqueWhite
Aqua
Aquamarine
Azure
Beige
Bisque
Black
BlanchedAlmond
Blue
BlueViolet
Brown
BurlyWood
CadetBlue
Chartreuse
Chocolate
Coral
CornflowerBlue
Cornsilk
Crimson
Cyan
DarkBlue
DarkCyan
DarkGoldenrod
DarkGray
DarkGreen
DarkGrey
DarkKhaki
DarkMagenta
DarkOliveGreen
DarkOrange
DarkOrchid
DarkRed
DarkSalmon
DarkSeaGreen
DarkSlateBlue
DarkSlateGray
DarkSlateGrey
DarkTurquoise
DarkViolet
DeepPink
DeepSkyBlue
DimGray
DimGrey
DodgerBlue
FireBrick
FloralWhite
ForestGreen
Fuchsia
Gainsboro
GhostWhite
Gold
Goldenrod
Gray
Green
GreenYellow
Grey
Honeydew
HotPink
IndianRed
Indigo
Ivory
Khaki
Lavender
LavenderBlush
LawnGreen
LemonChiffon
LightBlue
LightCoral
LightCyan
LightGoldenrod
LightGoldenrodYellow
LightGray
LightGreen
LightGrey
LightPink
LightSalmon
LightSeaGreen
LightSkyBlue
LightSlateBlue
LightSlateGray
LightSlateGrey
LightSteelBlue
LightYellow
Lime
LimeGreen
Linen
Magenta
Maroon
MediumAquamarine
MediumBlue
MediumOrchid
MediumPurple
MediumSeaGreen
MediumSlateBlue
MediumSpringGreen
MediumTurquoise
MediumVioletRed
MidnightBlue
MintCream
MistyRose
Moccasin
NavajoWhite
Navy
NavyBlue
OldLace
Olive
OliveDrab

Orange
OrangeRed
Orchid
PaleGoldenrod
PaleGreen
PaleTurquoise
PaleVioletRed
PapayaWhip
PeachPuff
Peru
Pink
Plum
PowderBlue
Purple
Red
RosyBrown
RoyalBlue
SaddleBrown
Salmon
SandyBrown
SeaGreen
Seashell
Sienna
Silver
SkyBlue
SlateBlue
SlateGray
SlateGrey
Snow
SpringGreen
SteelBlue
Tan
Teal
Thistle
Tomato
Turquoise
Violet
VioletRed
Wheat
White
WhiteSmoke
Yellow
YellowGreen

Duplicate colors: *Aqua* = *Cyan*, *Fuchsia* = *Magenta*; *Navy* = *NavyBlue*; *Gray* = *Grey*, *DarkGray* = *DarkGrey*, *LightGray* = *LightGrey*, *SlateGray* = *SlateGrey*, *DarkSlateGray* = *DarkSlateGrey*, *LightSlateGray* = *LightSlateGrey*, *DimGray* = *DimGrey*.

HTML4 color keyword subset: *Aqua*, *Black*, *Blue*, *Fuchsia*, *Gray*, *Green*, *Lime*, *Maroon*, *Navy*, *Olive*, *Purple*, *Red*, *Silver*, *Teal*, *White*, *Yellow*.

Colors taken from Unix/X11: *LightGoldenrod*, *LightSlateBlue*, *NavyBlue*, *VioletRed*.

4.4 Colors via `x11names` option

AntiqueWhite1
AntiqueWhite2
AntiqueWhite3
AntiqueWhite4
Aquamarine1
Aquamarine2
Aquamarine3
Aquamarine4
Azure1
Azure2
Azure3
Azure4
Bisque1
Bisque2
Bisque3
Bisque4
Blue1
Blue2
Blue3
Blue4
Brown1
Brown2
Brown3
Brown4
Burlywood1
Burlywood2
Burlywood3
Burlywood4
CadetBlue1
CadetBlue2
CadetBlue3
CadetBlue4
Chartreuse1
Chartreuse2
Chartreuse3
Chartreuse4
Chocolate1
Chocolate2
Chocolate3
Chocolate4
Coral1
Coral2
Coral3
Coral4
Cornsilk1
Cornsilk2
Cornsilk3
Cornsilk4
Cyan1
Cyan2
Cyan3
Cyan4
DarkGoldenrod1
DarkGoldenrod2
DarkGoldenrod3
DarkGoldenrod4
DarkOliveGreen1
DarkOliveGreen2
DarkOliveGreen3
DarkOliveGreen4
DarkOrange1
DarkOrange2
DarkOrange3
DarkOrange4
DarkOrchid1
DarkOrchid2
DarkOrchid3
DarkOrchid4
DarkSeaGreen1
DarkSeaGreen2
DarkSeaGreen3
DarkSeaGreen4
DarkSlateGray1
DarkSlateGray2
DarkSlateGray3
DarkSlateGray4
DeepPink1
DeepPink2
DeepPink3
DeepPink4
DeepSkyBlue1
DeepSkyBlue2
DeepSkyBlue3
DeepSkyBlue4
DodgerBlue1
DodgerBlue2
DodgerBlue3
DodgerBlue4
Firebrick1
Firebrick2
Firebrick3
Firebrick4
Gold1
Gold2
Gold3
Gold4
Goldenrod1
Goldenrod2
Goldenrod3
Goldenrod4
Green1
Green2
Green3
Green4
Honeydew1
Honeydew2
Honeydew3
Honeydew4
HotPink1
HotPink2
HotPink3
HotPink4
IndianRed1
IndianRed2
IndianRed3
IndianRed4
Ivory1
Ivory2
Ivory3
Ivory4
Khaki1
Khaki2
Khaki3
Khaki4
LavenderBlush1
LavenderBlush2
LavenderBlush3
LavenderBlush4
LemonChiffon1
LemonChiffon2
LemonChiffon3
LemonChiffon4
LightBlue1
LightBlue2
LightBlue3
LightBlue4
LightCyan1
LightCyan2
LightCyan3
LightCyan4
LightGoldenrod1
LightGoldenrod2
LightGoldenrod3
LightGoldenrod4
LightPink1
LightPink2
LightPink3
LightPink4
LightSalmon1
LightSalmon2
LightSalmon3
LightSalmon4

xcolor v2.11 (2007/01/21) 40

LightSkyBlue1
LightSkyBlue2
LightSkyBlue3
LightSkyBlue4
LightSteelBlue1
LightSteelBlue2
LightSteelBlue3
LightSteelBlue4
LightYellow1
LightYellow2
LightYellow3
LightYellow4
Magenta1
Magenta2
Magenta3
Magenta4
Maroon1
Maroon2
Maroon3
Maroon4
MediumOrchid1
MediumOrchid2
MediumOrchid3
MediumOrchid4
MediumPurple1
MediumPurple2
MediumPurple3
MediumPurple4
MistyRose1
MistyRose2
MistyRose3
MistyRose4
NavajoWhite1
NavajoWhite2
NavajoWhite3
NavajoWhite4
OliveDrab1
OliveDrab2
OliveDrab3
OliveDrab4
Orange1
Orange2
Orange3
Orange4
OrangeRed1
OrangeRed2
OrangeRed3
OrangeRed4
Orchid1
Orchid2
Orchid3
Orchid4
PaleGreen1
PaleGreen2
PaleGreen3
PaleGreen4
PaleTurquoise1
PaleTurquoise2
PaleTurquoise3
PaleTurquoise4
PaleVioletRed1
PaleVioletRed2
PaleVioletRed3
PaleVioletRed4
PeachPuff1
PeachPuff2
PeachPuff3
PeachPuff4
Pink1
Pink2
Pink3
Pink4
Plum1
Plum2
Plum3
Plum4
Purple1
Purple2
Purple3
Purple4
Red1
Red2
Red3
Red4
RosyBrown1
RosyBrown2
RosyBrown3
RosyBrown4
RoyalBlue1
RoyalBlue2
RoyalBlue3
RoyalBlue4
Salmon1
Salmon2
Salmon3
Salmon4
SeaGreen1
SeaGreen2
SeaGreen3
SeaGreen4
Seashell1
Seashell2
Seashell3
Seashell4
Sienna1
Sienna2
Sienna3
Sienna4
SkyBlue1
SkyBlue2
SkyBlue3
SkyBlue4
SlateBlue1
SlateBlue2
SlateBlue3
SlateBlue4
SlateGray1
SlateGray2
SlateGray3
SlateGray4
Snow1
Snow2
Snow3
Snow4
SpringGreen1
SpringGreen2
SpringGreen3
SpringGreen4
SteelBlue1
SteelBlue2
SteelBlue3
SteelBlue4
Tan1
Tan2
Tan3
Tan4
Thistle1
Thistle2
Thistle3
Thistle4
Tomato1
Tomato2
Tomato3
Tomato4
Turquoise1
Turquoise2
Turquoise3
Turquoise4
VioletRed1
VioletRed2
VioletRed3
VioletRed4
Wheat1
Wheat2
Wheat3
Wheat4
Yellow1
Yellow2
Yellow3
Yellow4
Gray0
Green0
Grey0
Maroon0
Purple0

Duplicate colors: *Gray0* = *Grey0*, *Green0* = *Green1*.

Bibliography

[1] Robert Bozick. Making it through the first year of college: The role of students' economic resources, employment, and living arrangements. *Society of Education*, 80:261–285, July 2007.

[2] *The Matrix Reference Manual*, 2005.

[3] DongbinKim. The effect of financial aid on students' college choice: Differences by racial groups. *Research in Higher Education*, 45(43-70), February 2004.

[4] Alicia C Dowd and Tarek Coury. The effect of loans on the persistence and attainment of community college students. *Research in Higher Education*, 47:33–62, February 2006.

[5] Susan M Dynarski. Does aid matter? measuring the effect of student aid on college attendance and completion. NBER Working Paper No, 7422, November 1999.

[6] Hans Johnson, Marisol Cuellar Mejia, David Ezekiel, and Betsey Zeiger. Student debt and the value of a college degree. Public Policy Institute of California website, San Francisco, CA, June 2013.

[7] Robert J Manthei and Alison Gilmore. The effect of paid employment on university studentsâĂŹ lives. *Education and Training*, 47(3):202–215, 2005.

[8] Michael S. McPherson and Morton O. Schapiro. Does student aid affect college enrollment? new evidence on a persistent controversy. *The American Economic Review*, 81:309–318, March 1991.

Index