Poisson Kernel Estimates of Lexigraphic Matrices

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Abstract
In this study we examine the suitability of efficiently inverting, or obtaining the solution of problems associated with a class of centrosymmetric matrices which arise from problems associated with stochastic interpolation on regularly spaced grids. These matrices associated with these linear systems are full, and thus the usual range of sparse matrix solvers are not available. In particular, the study focuses directly on the properties of centrosymmetric matrices as a means to improve the solvability of these linear systems, showing that for these centrosymmetric matrices of even order it is possible to achieve a factor of 7/8 improvement in the cost of matrix inversion using Schur complements. In addition the multiplication of these matrices can be speeded up by a factor of two by fully utilizing the block antisymmetry of these matrices.

1. Background

A square matrix has the same number of rows as it does columns, and it represents a linear system in which the number of unknowns exactly matches the number of equation, and thus it represents a linear system which may have a unique solution. The size of an $n \times n$ matrix is referred to as the order of the matrix n, and this equals the number of rows or columns in a square matrix.

Finally, in terms of notation, it will be convenient to refer to the matrix A in terms of its coefficients, using the notation $A = (a_{ij})$. For example, if we multiply the matrix A by the constant c, then $cA = c(a_{ij}) = (ca_{ij})$ and we demonstrate through this notational device that matrix multiplication is equivalent to multiplying each entry in the matrix by the constant c. This standard notational tool will prove useful in regard to demonstrating some properties of matrices that are needed in this study.

2. A special matrix of interest: a centrosymmetric matrix

A matrix with a particular structure, or a specific form associated with the pattern of the coefficients is called a *special matrix*. A special matrix need not be complicated, as in the case of the diagonal matrix already discussed, or it can be much more intricate in the layout of its coefficients in some regular pattern. The primary focus of this study is on centrosymmetric matrices, although even at this level of specialization, not every centrosymmetric matrix will be of interest to us. The centrosymmetric matrices that are of interest are those which arise from problems associated with stochastic interpolation.

Centrosymmetric matrices[1] follow a pattern in which the coefficients of the matrix are reflected about the center of the matrix [2], i.e., the coefficients of the matrix A are rotationally symmetric about the center. Alternatively, we can define a centrosymmetric matrix such that $A = EAE$, where $E = (\boldsymbol{e}_n, \boldsymbol{e}_{n-1}, \ldots, \boldsymbol{e}_1)$ is the exchange matrix, and $\boldsymbol{e}_k$ is the k-th standard basis vector in $\mathbb{R}^n$. In this description, the coefficients of the matrix progress lexicographically for the first half of the rows in the matrix, and then regress through these same coefficients listed in reverse lexicographic order from the bottom row to the middle row, if the matrix is of even order. If it is of odd order, the middle row is symmetric about its center. Perhaps the easiest way to conceptualize this pattern is through language using palindromes. When read backwards, a palindrome produces the same sentence when read both forwards and backwards as in the sentence, “Go hang a salami, I’m a lasagna hog.”

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3. Developments

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[1] These are sometimes referred to as perplectic matrices.

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i	1	2	3	4	5
P_i	0.34	0.11	0.11	0.11	0.11 1
$l(P)$	1	2	2	2	2
code(P)	0	10	11	12	20
Ideal Length	0.982	2.009	2.009	2.009	2.009

Table 1: Tabulated results in base 3 with actual and ideal code length.

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4. Introduction to technical writing

This is the introduction section. This should involve a brief discussion of the purpose of the report, what concepts are essential, and what will be investigated. Use this template for writing your analysis files. If you examine this file closely, it has some very useful features in it that will set up your paper so that it has a more professional appearance. You should also review the rules for typesetting LaTeX. Be mindful of some simple rules in using English:

1. Do not use the first person singular in a formal report. Use the first person plural or the third person in a passive construction. Thus

   I did this work, is unacceptable, while, *We did this work*, or *This work was done*, are acceptable.

2. When typesetting mathematics, use punctuation. All math equations must read as if they were English sentences. You must end these with a period, and put commas where needed.

3. Keep the verb tense constant. Do not switch from past to present, and then back again, and then back again to the present. There may be a need to do this, however be careful when you are doing it.

4. Avoid colloquial English. These are formal reports. These are not email messages to your friends. Be aware that some words in English such as *big* are colloquial, and do not appear in formal writing. Do not use *very* to modify every adjective. It is usually meaningless.

   Get a manual on English usage and style from the library, or purchase one from the bookstore. Realize that if you plan to stay in academia, or get an administrative position in a company or a laboratory, you have to be able to communicate effectively. Work on this skill.

Then be mindful of some simple rules for typesetting LaTeX:

1. Note that the previous sentence beginning this list is not indented since it does not begin a new paragraph. To avoid indentation, use the `\noindent` command.

2. Note the `\usepackage` command which changes the font and invokes the AMS mathematics commands (which simplify typesetting). Note the use of the
   `\thispagestyle{empty}`
   command to avoid the printing of page number on the title page.

3. Use LaTeXcommands carefully. Please review how to use math mode. Many of the reports had math in the text that was not in math mode. All symbols, variable, and so on are to be written in math mode. We do not use computer science variables to write equations, either, thus

   $$\sum_{i=1}^{upperbound} x_i^2$$

   is not acceptable, while in contrast,

   $$\sum_{i=1}^{n} x_i^2$$

   is acceptable.

4. There is no need for asterices in multiplication. Thus ab or xy, not $a*b$ or $x*y$.

5. Please note that LaTeXuses the forward and backward quotes. Thus we write *“This is a quote”*, and not *”This is a quote”*. Examine this example closely. The forward quote is written using the ‘ and not the ’ quote. The forward quote is typically on the upper left of your keyboard, on the tilde key, and below the Esc key.

4.1 Background on technical writing

Introduce the mathematical ideas and how they are implemented in this section. You should not have any results here. Only introduce the mathematics that is necessary to discuss your results. For example, the method used to compute each norm may be important, however you may choose to either typeset the equations or refer to these in your text, or any other reference. You may refer to equations using a bibliography reference generated using a bib file.

4.2 What's in the results

Discuss the results. This means that you provide details concerning the numerical experiments that were performed. This section should include graphs, tables and other supporting documentation. It should read well, and all items discussed in the text should refer to tables, graphs and equations using the reference command in LaTeXand using the format `Table~\ref{table:mytable}`, `Figure~\ref{figure:myfigure}` to refer to tables and figures. Equations should be referred to using the number of the equation in parentheses, i.e., `(\ref{eq:myequation})`. You never use the word 'equation', except at the beginning of a sentence, in which case you use the construction: `Eq.~(\ref{eq:myequation})`. Note that you need the `~`, i.e., the tilde, to avoid having LaTeXbreak the object across a line. Always tie counters to their attributes using a tilde or a thin space, e.g.,

tied using a tilde: `in Sec.~\ref{sec:topics}` $\Longrightarrow$ in Sec. 3.7

tied using a thin space: `in Sec.\,\ref{sec:topics}` $\Longrightarrow$ in Sec. 3.7

untied (wrong): `in Sec. \ref{sec:topics}` $\Longrightarrow$ in Sec. 3.7

which ties the object Sec. to the counter. Not only is the space too large after the period when there is no tie, but there is the danger that the word, Sec., will go at the end of one line, and the number 3.7 at the start of a new line.

When doing numerical studies, you must present the results of all of your numerical investigations. For the case of polynomial interpolation, at the very least these results should have examined the behaviour of the polynomials interpolated on 2^k points, where $k = 2, 3, \ldots, n$, where n is sufficiently large so as to be able to determine numerically whether convergence was achieved. A value of $n = 5$, or $n = 10$ is usually insufficient.

Focus on the error. Numerical methods are approximations. They introduce error. Examine the errors, and attempt to understand where they are coming from. Examine issues of computational efficiency, and other topics as appropriate. Time your results when you run the codes. You can always choose not to use the timing results, however if they show something important, then you can use it.

4.3 What's in the conclusion

Be brief, however one sentence is not sufficient. Try to concisely summarize what was discussed in the results. Avoid rewriting the results section. Someone reading your report should be able to understand the writeup from what is written in this section. This is one of the most important parts of your report (and probably the only one your supervisor or manager will ever read, so you might as well learn to write it effectively).

5. Using figures

We demonstrate a technique to set up and configure a figure using the `\includegraphics` command embedded into the picture environment embedded into the figure environment. The picture environment is needed to control the spacing, while the figure environment is needed to control the floating of the figures in the document. Note that you can refer to the figure with its label, i.e., in Figure 2 you have the setup of the figure. Note that you need to create the figures so that the labels and legends scale well with the fonts being used after you have scaled the figure. The examples in Fig. 1 and Fig. 2 should be examined in detail as the axes labels are not generated with the figures. Instead, they are added as overlays using the `\put` command in the `picture` environment.

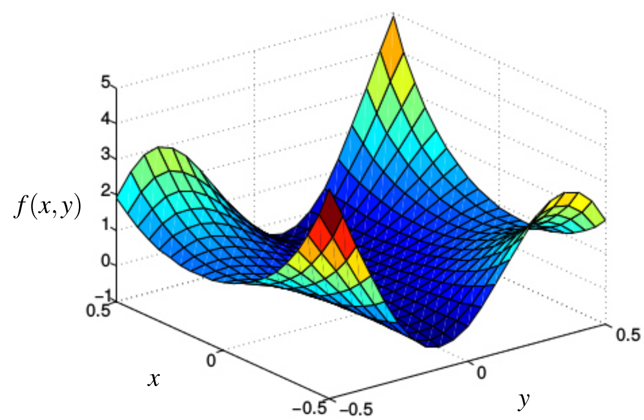

Figure 1: Example of using a raster image which is scaled and annotated with axes.

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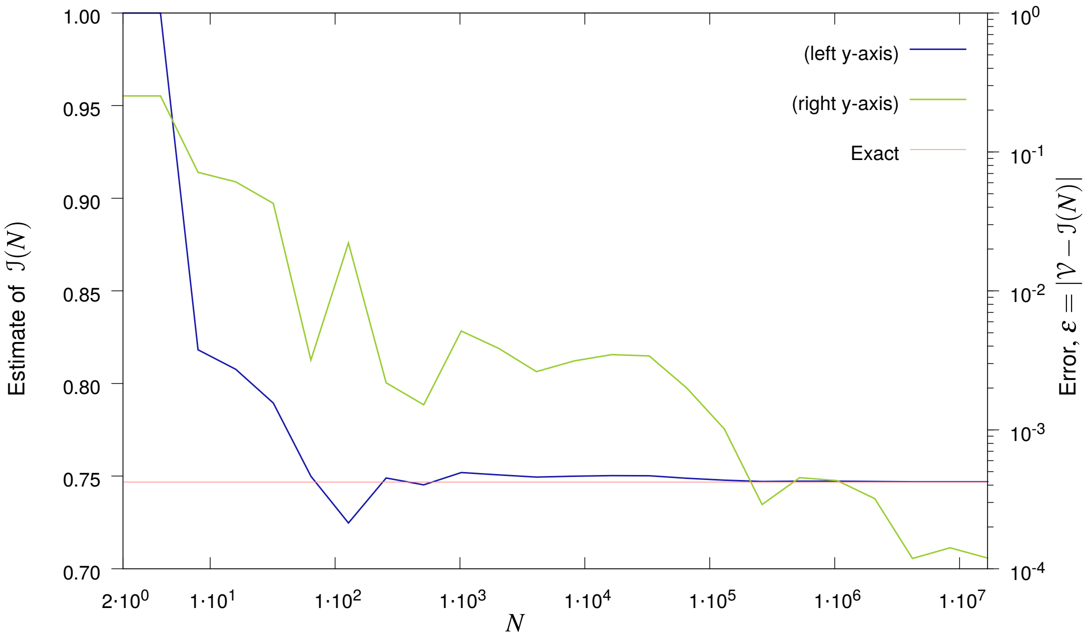

Figure 2: An approximation $\mathcal{I}(N)$ to the value of the integral for $\int_0^1 e^{x^2}\,dx$ obtained using a Monte Carlo approach. The estimate for $\mathcal{I}(N)$ is a function of the number of samples N, and the value of $I \approx 0.7468241328 \equiv \mathcal{V}$ was obtained using Wolfram Alpha®.

5.2 Using another figure

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5.3 Even more results

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6. Conclusions

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Where can we find information about this thesis style? Go to `http://math.newhaven.edu/mathphysics`.

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Acknowledgments

I would like to thank my advisor Dr. Strange Strangelove for guidance throughout the project, and Dr. Tom Clown for the help with everything else, and everyone else for financial support.

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She has published one student article and has given two student presentations during the May, 2014 MAA Conference at UNCC, Topeka, AZ.